27 October 2006
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S = compact, oriented smooth, $\partial = \emptyset$, $d-1$ dim manifold
(not nec. connected)

$$\dot{m}_S = \{ \text{set } M \text{ a } d\text{-manifold s.t. } \partial M = S \}$$

$$\begin{tikzcd} M & \\ & S \arrow[ul] \arrow[dl] \\ M' \arrow[uu, "\text{diffeo}"] & \end{tikzcd}$$

$M \equiv M'$
$\exists$ diffeo extending id on ∂

$$m_S = \mathbb{C}[\dot{m}_S] \ni \sum a_i M_i, \text{ finite sum.}$$

Universal (S) pairing

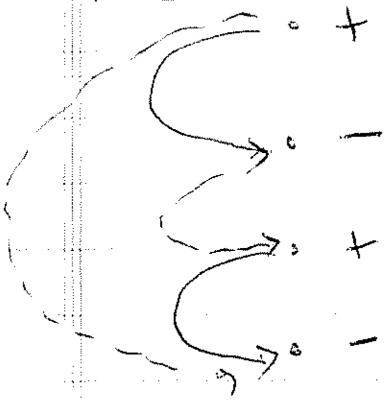
$$m_S \times m_S \xrightarrow{\text{glue}} \underbrace{m_\emptyset = m^d}_{\mathbb{C}\text{-span of closed } d\text{-manifolds}}$$

$$\sum a_i M_i \times \sum b_j N_j \to \sum a_i \bar{b}_j M_i \bar{N}_j$$

complex conjugation

reversal of orientation

$\boxed{d=1}$

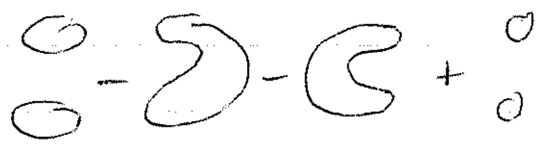


M (red)

N (green)

$$M\bar{N} = S^1$$

$$\langle M-N, M-N\rangle = M\bar{M} - M\bar{N} - N\bar{M} + N\bar{N}$$



$$= 2(S^1 \amalg S^1) - 2(S^1)$$

$$\begin{tikzcd}
\mathcal{M}_s \times \mathcal{M}_s \arrow[r] \arrow[d] & \mathcal{M} \arrow[d, "Z"', "\text{action}"] \\
V_Z \times V_Z \arrow[r] & \mathbb{C}
\end{tikzcd}$$

divide by nullity

$d=3$, action $= C.S.(a)$

$$Z = \int \mathcal{D}a \, e^{-2\pi i k \, C.S.(a)}$$

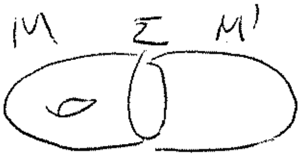
$$C.S(a) = \frac{1}{8\pi^2}\int a \wedge da + \frac{2}{3} a \wedge a \wedge a$$

$((d-1)+1)$ - U TQFT $\qquad d-1=2, \quad d=2+1.$

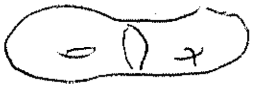
1. closed 3-manifold $M \longrightarrow Z(M) \in \mathbb{C}$.
2. 2-manifold $\Sigma \longrightarrow V(\Sigma)$ and


$\Rightarrow v(M) \in V(\Sigma)$

3. gluing


$M \cup \bar{M}'$ or $M\bar{M}'$

$$Z(M\bar{M}') = \langle v(M), v(M') \rangle$$

$V($  $) = \sum_{\text{labels}} V($  $) \otimes V($  $)$

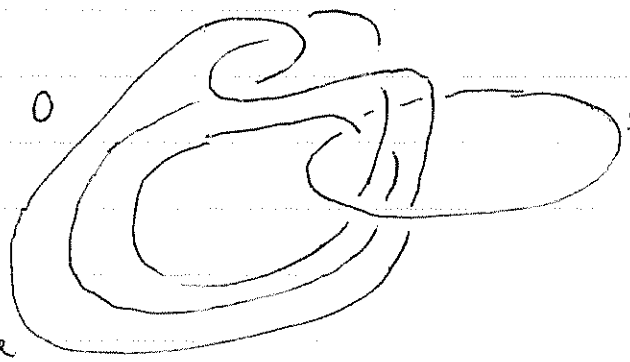
$V \subset \mathcal{M}_s$ s.t. $ZP(v,w)=0 \ \forall w$.

$$\mathcal{M}_s \times \mathcal{M}_s \xrightarrow{P} \mathcal{M}$$

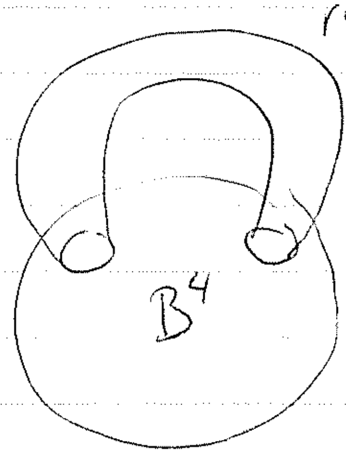
digression $d=4$ $(d=1=3)$

Mazur manifold H_* 

Kirby link calculus:



$$\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

$\partial\Bigg($  $\Bigg) = S^1 \times S^2$

$$\Big(D^2 \times D^2,\ \partial\underbrace{(D^2 \times D^2)}_{\text{solid torus}}\Big)$$

For the Mazur manifold M

$$\langle M| \in \mathring{\mathcal{M}}_{\text{Mazur 3-sphere}}$$

$$\langle M'| \in$$

Wang, Slingerland, Walker, Kitaev, Nayak

$$\langle M-M', M-M' \rangle = S^4 - S^4 - S^4 + S^4 = 0.$$

1980's Akbulut

$K(3)$ log trans $(K(3))$

Walker, Calegari

d	1	2	3	4	5	6	7	8
	Positive		Positive	nullity	Teichner Kreck nullity			

$E_8 + E_8$ vs. E_{16}

$$H_*(K(3);\mathbb{Z}) = E_8 + E_8 + 3\begin{vmatrix}0&1\\1&0\end{vmatrix}$$

$$\langle (E_8+E_8) - E_{16} | \in \mathcal{M}_{S^3}$$

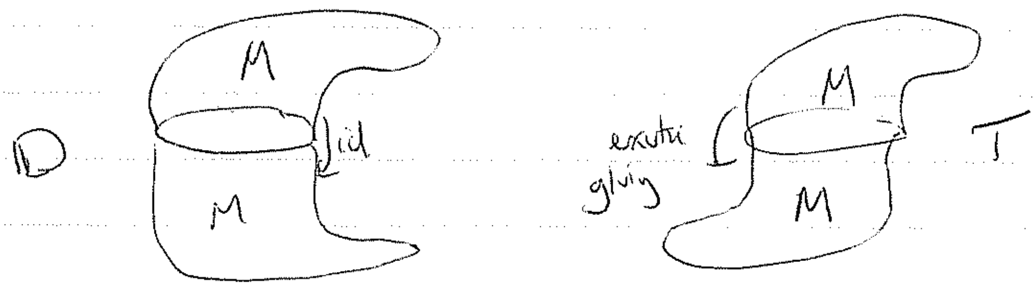
$$E_{16} \oplus -E_{16} \cong 16\begin{pmatrix}0&1\\1&0\end{pmatrix}$$

UTQFT's

C. Matter H's

In UTQFT, $\langle v,v\rangle = 0 \Rightarrow v = 0$.

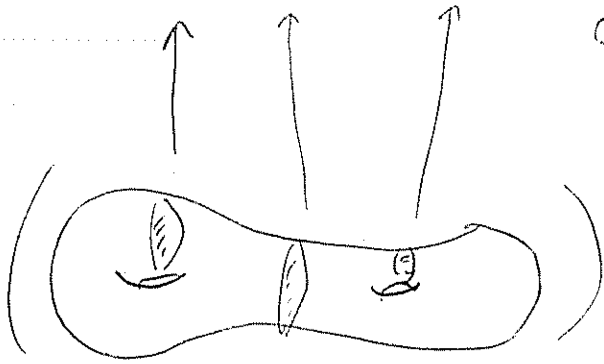
$d=4$ null vector $\Leftarrow$ "double" = "twisted double"



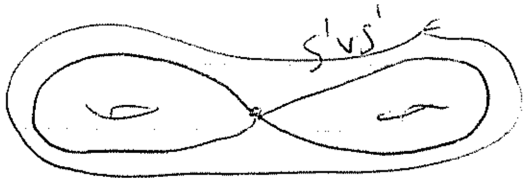
$$\langle M - M', M - M'\rangle = 0$$

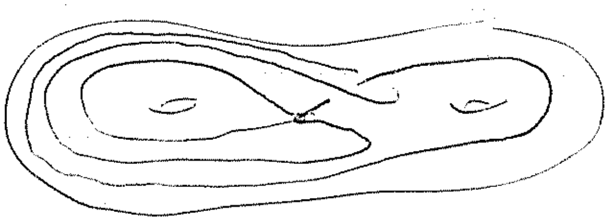
$S^1\times S^2 \# S^1\times S^2 \setminus S^1\vee S^1$

$\simeq id$



D





$2 = S^1\times D^2 \natural S^1\times D^2$

$=$ 

$\Sigma = S^1\times S^2 \# S^1\times S^2$

$+ = \natural$ or $\#$

$2 + X + 2$

glue $2 + X + 2$

double $\overline{2} + DX + \overline{2} = DX + \overline{4}$

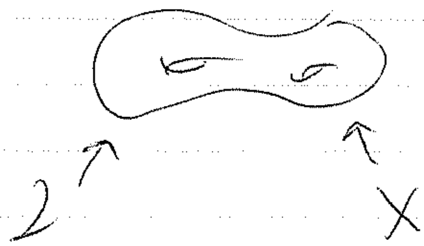
$2 + X + 2$

glue $X + 2 + 2$

twist $\overline{2} + \overline{2}$

Is it possible for $2+2 = DX$? (in red)

$V(\overline{2})$ $\qquad g(\overline{2}) = 2$



V be +-def. Hermitian (Euclidean)

$V(2) \neq V(X)$

$\langle v(2), v(2)\rangle = Z(\overline{2})$ $\quad \Rightarrow |v(x)| > |v(2)|$

$\langle v(2), v(X)\rangle = Z(\overline{2})$ $\quad \Rightarrow Z(D(X)) = \langle v(x), v(x)\rangle > Z(\overline{2})$