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Reference | K. Gawedki
in Quantum Fields & Strings, vol. II
lecture 3: σ-models

Ricci Flow & Renormalization

Classical Physics: φ_{cl}, $S[\varphi_{cl}]$. $\delta S = 0 \longleftrightarrow$ physical states of theory

Quantum Physics: again, have $S[\varphi]$

$$\langle F \rangle = \int_{Maps(\Sigma, M)} F(\varphi)\, e^{-\frac{1}{\hbar} S(\varphi)} \underbrace{D\varphi}_{\prod_{x \in \Sigma} d\varphi_x}$$

Σ = worldline, M = spacetime

more generally, $\dim \Sigma = d+1$

eg $S(\varphi) = \frac{\beta}{4\pi} \int_\Sigma (|d\varphi|^2 + m^2 \varphi^2)\, dv = \frac{1}{2}(\varphi, G^{-1}\varphi)_{L^2}$

$$\frac{\beta}{2\pi} G = (-\Delta + m^2)^{-1}$$

$$Z = \int e^{-S(\varphi)} D\varphi = \int e^{-\frac{1}{2}(\varphi, G^{-1}\varphi)} D\varphi$$

if finite dim'l, answer $(\det G)^{1/2}$

Regularization Scheme ζ-fn regularization

$\lambda_1, \lambda_2, \lambda_3, \ldots$ eigenvalues of G

λ_n grows as $\mathcal{O}(n^{-2/d+1})$

$\zeta_G(s) = \sum \lambda_n^{-s}$ converges for $Re(s) \geq \ldots$

analytically continue to $s=0$

$\zeta_G'(0) = \log \det G$

focus on $d+1=2$ $\Sigma \to M$ = Riemannian mfld g_{ij}

$\varphi \in Map(\Sigma, M)$

$$S(\varphi) = \frac{1}{4\pi} \|d\varphi\|^2_{L^2} = \frac{1}{2\pi} \int_\Sigma g_{ij}(\varphi)\, d\varphi^i\, d\varphi^j$$

often add other terms

<u>topological term)</u> $\frac{i}{4\pi}\int_\Sigma \varphi^*\omega$ $\quad\omega\in\Omega^2(M)$ (axion)
ignore for today

<u>tachyon)</u> $u: M\to\mathbb{R}$ $\quad \frac{1}{4\pi}\int_\Sigma u\circ\varphi \, dvol_\Sigma$

<u>dilaton)</u> $\omega: M\to\mathbb{R}$ $\quad \frac{1}{4\pi}\int_\Sigma \omega\circ\varphi \, R\, dvol_\Sigma$

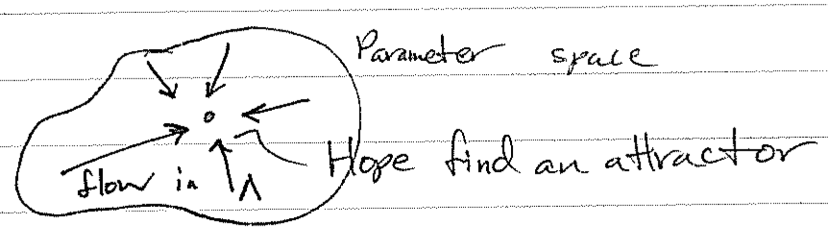
Want to do $\displaystyle\int_{Maps(\Sigma,M)} e^{-\frac{1}{\hbar}S(\varphi)}\, D\varphi$

Need: → Regularization scheme, → Renormalization

e.g. lattice approximation to Σ, put in Box $\quad \displaystyle\int_{Maps(\Sigma_{discrete},M)} S^{-\frac{1}{\hbar}S(\varphi)}\, D\varphi$

depends on distance scale of lattice: cutoff Λ

if consider {Family of theories, "constants" in S varied}
find $g_{ij}(\Lambda)$, $u(\Lambda)$, ...
s.t. $\lim_{\Lambda\to\infty}$ of $\int F e^{-\frac{1}{\hbar}S}\, D\varphi$ exist



Perturbation Expansion

role of $\hbar$] $\hbar \to 0$ limit: restore the classical theory

could hope to have $$\int_\Lambda e^{-\frac{1}{\hbar}S(\varphi)} D\varphi = F_{0,\Lambda} + F_{1,\Lambda}\hbar + \mathcal{O}(\hbar^2)$$

Outline] Regularization scheme : "dimensional regularization"

$\dim \Sigma = (d+1) + \varepsilon$, take $\varepsilon \to 0$

important, b/c many quantities have poles as $\dim \Sigma \to 2$

Renormalization scheme : "minimal subtraction"

$F_j(\varepsilon)\hbar^j$

$$F_j(\varepsilon) = \frac{1}{\varepsilon} F_j^{\text{pole}}(\varepsilon) + \mathcal{O}(1)$$

subtract off the polar term

1-loop term

computation of $F_1(\varepsilon)$ leads to Ricci flow equation
(2^{nd} & 3^{rd} order terms are 0 for supersymmetric case, but 4^{th} order $\neq 0$)

eventual result] $\delta g_{ij} = R_{ij}$, $\delta u = \frac{-1}{\varepsilon}\Delta_g u$