Robert Wälder:

* Singular Elliptic Genus of Normal surfaces

$M \qquad \hat{M}$ mirror mfld

topol. Mirror-sym. test

$$h^{p,q}(M) = h^{n-p,q}(\hat{M})$$

$\hat{M}$ has resolution (crepant) $X \xrightarrow{f} \hat{M}$

$$K_X = f^* K_{\hat{M}}$$

then $h^{p,q}(M) = h^{n-p,q}(X)$

we can always define hodge numbers

Z cplx variety

$H^i_{cpct}(Z, \mathbb{C})$

$\rightsquigarrow h^{p,q}(Z)$ such that

$$h^{p,q}(Z \setminus W) = h^{p,q}(Z) - h^{p,q}(W)$$

(doesn't work w/ mirror sym. test)

<u>Batyrev</u>:

$$E(Z, u, v) = \sum h^{p,q}(Z)\, u^p v^q$$

$E_{\text{stringy}}(Z)$: $\quad X \xrightarrow{f} Z$

$$K_X - f^* K_Z = \sum_I a_i D_i$$

$$E_{str}(Z) = \sum_{J \subset I} E(D_J^\circ) \times \prod \frac{uv-1}{(uv)^{a_j+1}-1}$$

$G \curvearrowright V \qquad V/G$, crepant resol. $X \xrightarrow{f} V/G$ s.t. $K_X = f^* K_{V/G}$

orbifold invariants of (V, G)

$\cong$ invariants of crepant resolution

Euler characteristic

$$e_{orb}(V, G) = \frac{1}{|G|} \sum_{gh=hg} e(V^{g,h})$$

$$\| $$

$$e(X)$$

$$\begin{array}{ccc} Y & \dashrightarrow & X \\ \downarrow & & \downarrow \text{ resolution} \\ V & \longrightarrow & V/G \end{array}$$

Elliptic Genus

X compl. man., cpt

TX = hol tan bundle

x_i chern roots

$$\int_X \prod_{TX} \frac{x_i \,\Theta\left(\frac{x_i}{2\pi i} - z, \tau\right)}{\Theta\left(\frac{x_i}{2\pi i}, \tau\right)}$$

$\Theta(t,\tau)$ = Jacobi Θ-fct

$$\sim \sin(\pi t) \cdot \prod \left(1-q^n e^{2\pi i t}\right)\left(1-q^n e^{-2\pi i t}\right)$$

$$q = e^{2\pi i \tau}$$

Singular elliptic genus

(Borisov, Libgober, Chin-lung Wang)

$$X \xrightarrow{f} Z \qquad K_X - f^* K_Z = \sum_I a_i D_i$$

$$\int_X \prod_{TX} \frac{x_i \,\Theta(x_i - z)}{\Theta(x_i)} \prod_I \frac{\Theta(D_i - (a_i+1)z)\,\Theta(z)}{\Theta(D_i - z)\,\Theta((a_i+1)z)}$$

$$D_i = c_1(\mathcal{L}_{D_i})$$

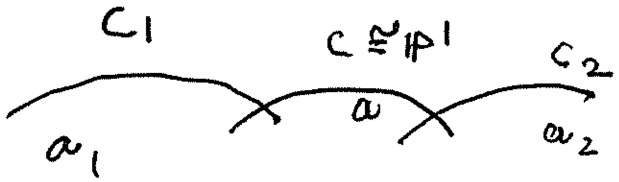
$= Ell_{sing}(Z)$

$$Ell_{sing}(V/G) = Ell_{orb}(V,G)$$

$a_i > -1$

$$K_X - f^*(K_Z + \varepsilon H)$$

Willem Veys $\rightarrow$ Normal surfaces singularities which are not purely log canonical



$C_1 \cap C = pt$

$C_2 \cap C = pt$

$C^0 = \mathbb{C}^*$

$uv = L$

$$\frac{(L-1)(L-1)}{(L^{a+1}-1)} + \frac{1\cdot(L-1)^2}{(L^{a_1+1}-1)(L^{a}-1)} + \frac{1\cdot(L-1)^2}{(L^{a_2+1}-1)(L^{a+1}-1)}$$

$$= \frac{(L-1)^2\,(L^{a_1+a_2+2}-1)}{(L^{a+1}-1)(L^{a_1+1}-1)\,(L^{a_2+1}-1)}$$

$$a_1 + a_2 + 2 = -C\cdot C\,(a+1)$$

$$= \frac{(L-1)^2(L^{(m-1)(a+1)}+\dots)}{(L^{a_1+1}-1)(L^{a_2+1}-1)}$$

$$\sum_{I} a_i D_i = \sum_{a_i=-1} -D_i + \sum_{a_i \neq -1} a_i D_i$$

$$\mathrm{Ell}_{\mathrm{sing}}(z) = \lim_{\varepsilon \to 0} \underset{a_i+\varepsilon b_i}{\mathrm{Ell}}\left(X, \sum (a_i+\varepsilon b_i) D_i\right)$$

$$\varepsilon \int \prod \frac{x_i \Theta(x_i - z)}{\Theta(x_i)} \frac{\Theta(E - \varepsilon z)}{\Theta(E - z)} \frac{\Theta(z)}{\Theta(\varepsilon z)}$$

$$\longrightarrow \int \prod \frac{x_i \Theta(x_i - z)}{\Theta(x_i)} \frac{\Theta(E + 2z)}{\Theta(E+z)} \frac{\Theta(z)}{\Theta(2z)}$$

$\to 0$