Allan Adams

"SU(3) mflds + Reid fantasy"

4d Physics $\longleftrightarrow$ 6 mfld

SUSY $\longleftrightarrow$ SU(3)

$\eta \neq 0 \quad \nabla\eta = 0$

massless fields $\longleftrightarrow$ moduli of SU(3)-str.

$X \quad \eta \neq 0 \implies \exists\, \nabla\eta = 0$

$J \neq 0 \qquad J_{\mu\nu} = \eta^{+} \Gamma_{\mu} \Gamma_{\nu} \eta \implies \overset{\mathbb{C}}{J}$ hermitian

$\nabla = \nabla_g + H$

- $H = 0 \implies dJ = 0$ CY
- $H \neq 0 \implies dJ \neq 0 \implies$ non-Kähler

$\mathcal{M}^{SU(3)}$

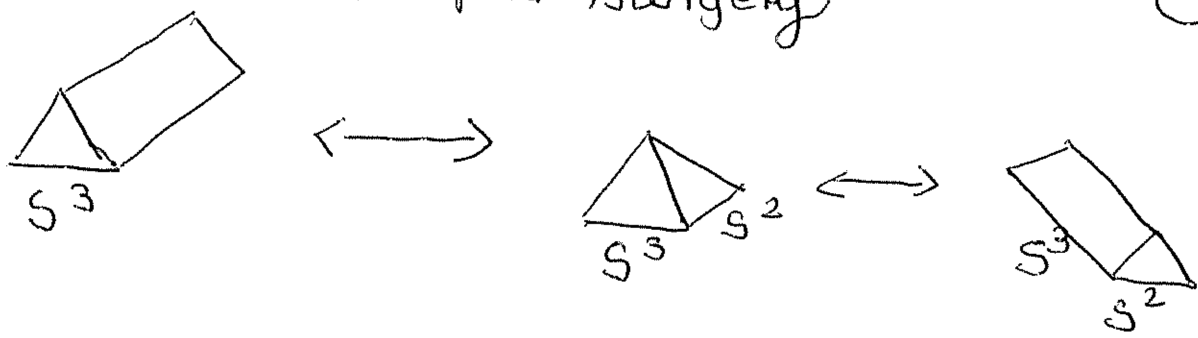
flop: when is it a smooth transition

$$(g + iB) = \mathcal{J}$$

(flop is smooth if you take B as add. structure)

Reid

conifold surgery

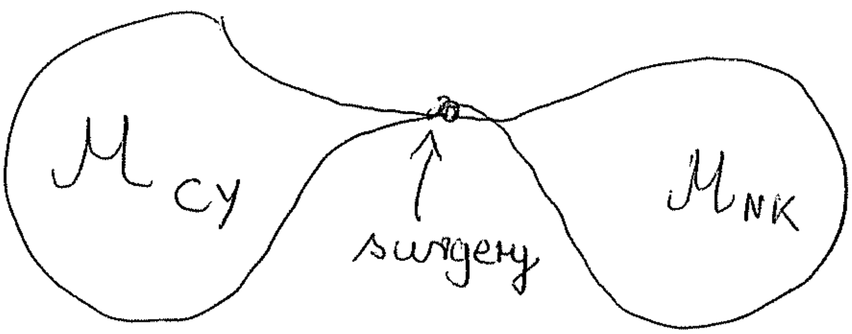


smooth transition for non-cpct space

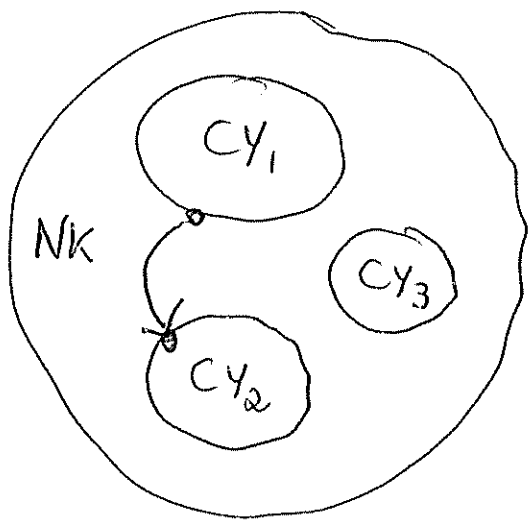
- cpct space CY M

take all $h^{1,1}$ $\mathbb{P}^1$'s $\longrightarrow 0$

conifold surgery $\Longrightarrow$ $\tilde{M}$ $\quad h^{1,1}_{\tilde{M}} = 0$ non-Kähler



Reid :



- is there always a path connecting CY-components (smooth in the sense of the conformal field theory) in heterotic string theory on X:

structure: J on X, vct-bdl $V \to X$
curvt. 2-form F

(in type II: Ric-flat)

here holom. vct-bdl V

first written out by Strominger

$$\begin{cases} F \wedge J \wedge J = 0 \\ H = i(\partial - \bar{\partial})J \neq 0 \\ d(\alpha\, J \wedge J) = 0 \\ dA = \operatorname{tr} R \wedge R - \operatorname{tr}(F \wedge F) \neq 0 \end{cases} \quad (+)$$

- in $H = 0 \rightsquigarrow$ CY-mflds

existence proof for $H \neq 0$
by Fu-Yau

$$\begin{tikzcd} T^2 \arrow[r, "\omega_i"] & X \arrow[d, "\pi\ \text{holom}"] & \pi^* V \arrow[l] \\ & S = K3 & V \arrow[l, "\text{holom}"'] \end{tikzcd}$$

holom vct

curvature of S^1-bdls

$$ds_X^2 = e^{u(s)}\, ds_S^2 + (d\theta_i + \alpha_i)^2$$

$$d\alpha_i = \omega_i$$

$$\Omega_X^{(3,0)} = \Omega_S^{(2,0)} \wedge \Theta^{(1,0)}$$

$$\Theta = (d\theta_1 + \alpha_1) + (d\theta_2 + \alpha_2)$$

$$H = (d\theta + \alpha_i) \wedge \omega_i$$
$$dH = \omega_i \wedge \omega_i \neq 0$$

defining data:

$[\omega_i] \in H^{1,1}(S)$

J_S on S

$\mathcal{V}_S \to S$

in this ansatz all eqns except (+) are satisfied:

existence $\Leftrightarrow$ existence of soln's for Eq. (+)

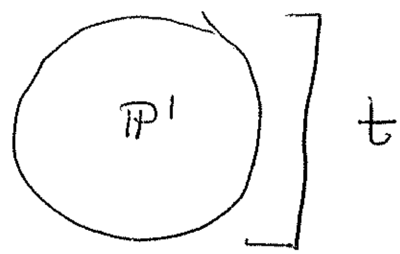
$$\begin{array}{ccc} \omega_1 & & \omega_2 \\ \downarrow & & \downarrow \end{array}$$
$$N_1^2 + N_2^2 = c_2(T_S) - c_2(\mathcal{V}_S)$$
$$= 24 - c_2(\mathcal{V}_S)$$

<u>now</u>:

$$ds_X^2 \to t^2 \, ds_X^2$$
$$J_X \to t^2 J_X$$
$$H \to t^2 H \quad N^2$$
$$R \wedge R \to t^0 \cdot N^4 \quad R \wedge R$$

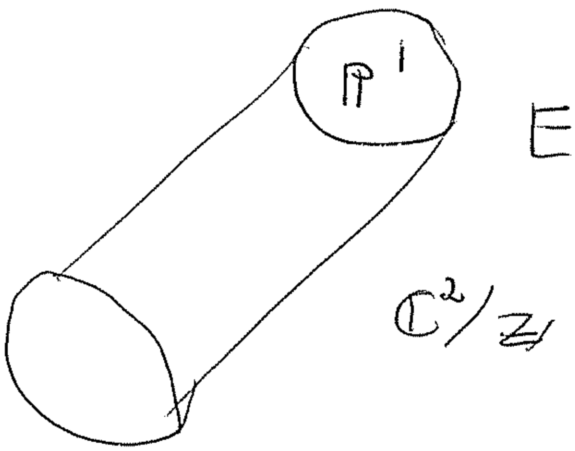
Ric-scalar $\sim t^{-2} N^2 \sim \mathcal{O}(1)$

$$\begin{tikzcd} S^1 \arrow[r] & S^3 \arrow[d] \\ & \mathbb{P}^1 \end{tikzcd}$$



non-cpt case:

$$\begin{tikzcd} T^2 \arrow[r, "\omega"'] & X \arrow[d] \\ & {} \end{tikzcd}$$



ODE in terms of $u(r)$

EH $\longleftarrow$ ν trivial

back to the K3-case

Gauged linear sigma model

2d gauge theory

$\mathcal{M}_H$

$F_a = [\phi_i ; \phi_j] \longleftrightarrow H_a \in H^2(\mathcal{M})$

$D\phi = 0$

find a gauge theory whose moduli space is CY - 3 fold

parameters in gauge theory
t^a

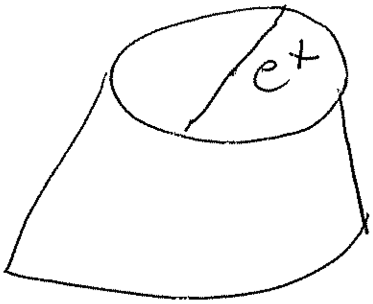
$\rightsquigarrow \quad J = t^a H_a$

$t \to \infty$ large radius $\rightsquigarrow$ CY

t^a

$t \to -\infty \longrightarrow$ Landau Ginzburg Orbifold

exact

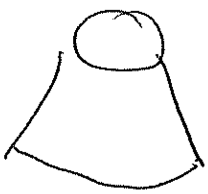


$\tilde{A}_1$

$\Big\downarrow \quad t \to -\infty$

$\mathbb{C}^2/\mathbb{Z}_2$ can be studied by means of CFT

you can find a gauge theory whose moduli space is

$T^2 \to X \to$ 

2d Gauge Theory $\underset{q_\mu}{(\lambda_-)}$ $\underbrace{(\phi, \psi_+)}_{Q_i}$

 $\sim \sum Q_i^2 - \sum q_\mu^2 = A$

$0 \overset{!}{=} c_2(T_{K3}) - c_2(V)$

Green - Schwarz - mechanism

Θ_i: 

$$\mathcal{L}_\Theta = (\partial\Theta + NA)^2 + N\Theta F$$

$$\xrightarrow{\alpha} \mathcal{L}_\Theta + \underline{\underline{N^2 \alpha F}}$$

cancels anomaly
from fermions

$$\left(T^2 \times_{\mathbb{Z}_2} \mathbb{C}^2\right)/\mathbb{Z}_2$$

$$Z = \sum_{gh} e^{\frac{i A g h \pi}{2}} \; \underset{g}{h\,\square}$$

$\int F = g$

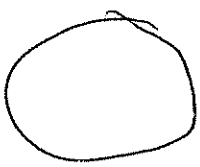
$\alpha = \frac{2\pi i h}{2}$

$$= \sum_{gh} (-1)^{\frac{A g h}{2}} \; \underset{g}{h\,\square}$$

$T^2 \times \mathbb{C}^2/\mathbb{Z}_2$, ν_1

$A = 0$

$H = 0 \quad dJ = 0$

 $\times \; \mathcal{M}_{T^2}$

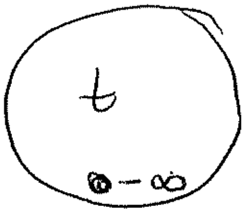
$T^2 \to \mathbb{C}^2/\mathbb{Z}_2$, ν_2

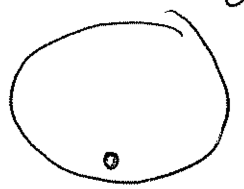
$A = 2$

$H \neq 0$ non-Kähler

b

at self-dual radius

$t = \mathrm{vol}(\mathbb{P}^1)$  $\times \; P_{SD}$

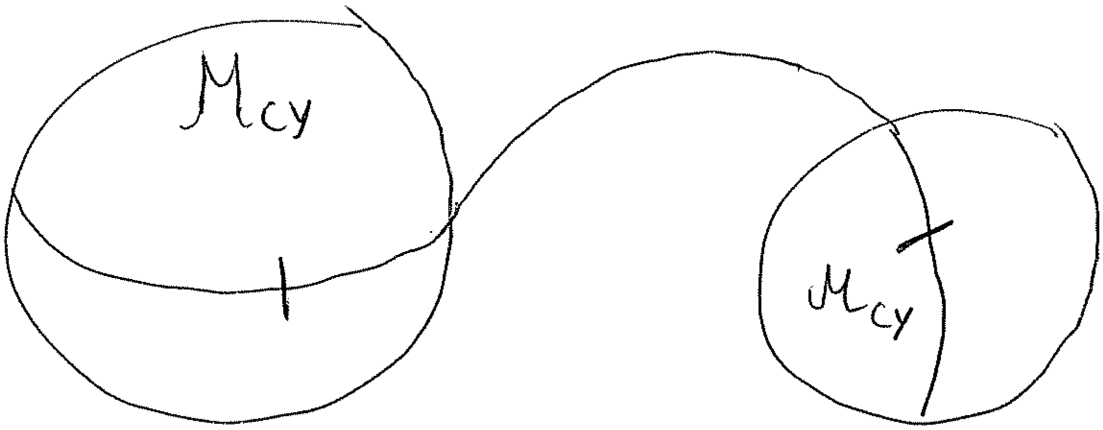
$t = \mathrm{vol}(S^3)$ 

$$\Theta \leftarrow A_{\Theta} = 0$$
$$\lambda_- \leftarrow A_{\lambda_-} = 2$$

$$A_{\Theta} = 2$$
$$A_{\lambda_-} = 0$$

at the two pts CFT's are isomorphic

$$\lambda_- \rightarrow e^{H}$$
$$e^{\Theta} \rightarrow \chi_-$$



* $u(1)$

	ϕ_1	ϕ_2
	1	-1

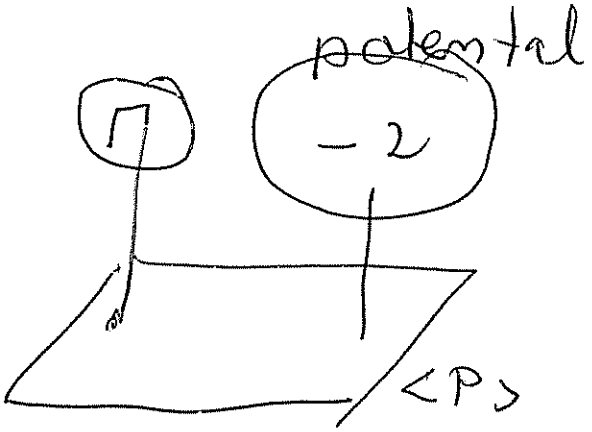
$$|(\partial + A)\phi_1|^2 + |(\partial + A)\phi_2|^2 + \big|\,|\phi_1|^2 - |\phi_2|^2 - t\,\big|$$

$\mathrm{Re}\, t \gg 1$

$\rightsquigarrow \ |\phi_1| \gg 1 \ \rightsquigarrow$ A massive; moduli space : $\mathbb{C}$

* $u(1)$

	ϕ_1	ϕ_2	P
	1	1	-2



$$(|\phi_1|^2 + |\phi_2|^2 - 2|P|^2 - 1)$$

$$|\phi_1|^2 + |\phi_2|^2 = P$$

$$S^3/u(1) = \mathbb{P}^1$$

$\Gamma \gg 1$

$$A(\phi) = \frac{\phi\, \partial\, \phi}{\Sigma |\phi|^2}$$

$$\mathcal{L} = |(\partial + A(\phi))\, \phi|^2$$

Eguchi – Hanson $\longrightarrow$ $\boxed{g_a(\phi)\ \partial\phi^i\ \partial\phi}$

$\Gamma \ll -1$ $\quad P \neq 0 \implies U(1) \downarrow \mathbb{Z}_2$

$$(\phi_1, \phi_2) \sim \mathbb{C}^2/\mathbb{Z}_2$$