29 February 2008

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SU(3) manifolds and the Reid fantasy

4d Physics $\longleftrightarrow$ 6 manifold

SUSY $\longleftrightarrow$ $SU(3)$ $\quad \eta \neq 0$

massless fields $\longleftrightarrow$ moduli of $SU(3)$ structure

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$X \quad \eta \neq 0 \Rightarrow \exists \ \nabla\eta = 0$

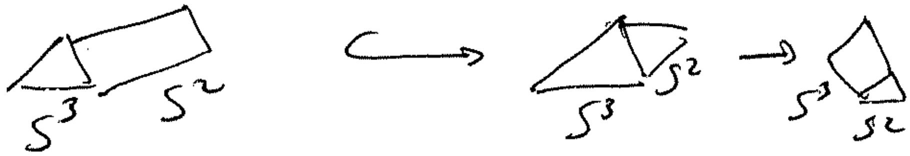
$J \neq 0 \quad J = \eta^{\dagger}\Gamma_{\mu}\Gamma_{\nu}\eta \Rightarrow \mathbb{C}$, J hermitian

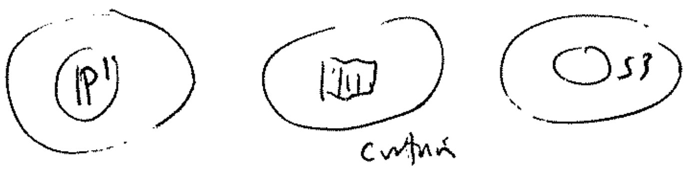
$\nabla = \nabla_g + H$

- $H = 0 \Rightarrow dJ = 0$ CY
- $H \neq 0 \Rightarrow dJ \neq 0 \Rightarrow$ non-Kähler

$\mathcal{M}^{SU(3)}$ flop $(g + iB) = \mathcal{J}$

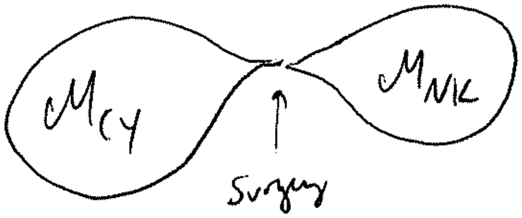
Reid

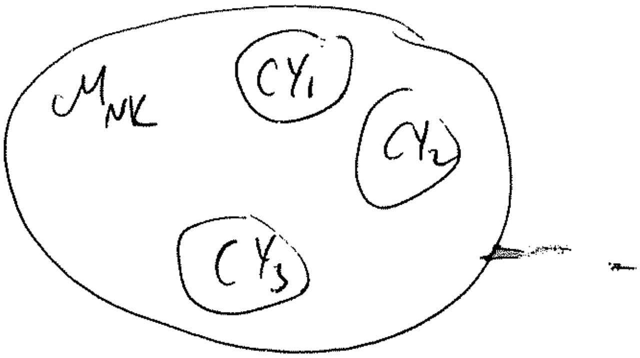




• CY M $\quad h^{21}\to 0 \quad \Rightarrow \quad \tilde{M} \quad h^{11}_{\tilde{M}}=0$

not Kähler (NK)





<u>Het. string on X</u>

$\exists\, X \qquad V\to X$

$R_{i\bar{\imath}}(X)=0 \qquad \mathrm{hol}(V)$ on X, $X = SU(3)$ manifold

$$F\wedge J\wedge J=0$$

$$H=i(\partial-\bar{\partial})J\neq 0$$

$$d(\alpha J\wedge J)=0 \qquad \text{"balanced"}$$

$$dH=\mathrm{tr}\,R\wedge R-\mathrm{tr}\,F\wedge F.$$

$$c_2(T_X)=c_2(V_X)$$

- $H = 0 \Rightarrow$ CY
- $H \neq 0$: Fu-Yau

$$\begin{tikzcd} T^2 \arrow[r] & X \arrow[d, "\pi"] & \mathcal{V}_X \arrow[l] \\ \omega_i & S & \mathcal{V}_S \arrow[l] \end{tikzcd}$$

$$ds^2_X = e^{u(s)} ds^2_S + (d\theta_i + \alpha_i)^2$$

$$d\alpha_i = \omega_i$$

$$\Omega_S \wedge \Theta \qquad \Theta = (d\theta_1 + \alpha_1) + (d\theta_2 + \alpha_2)$$

$$\Omega_X^{(3,0)} = \Omega_S^{(2,0)} \wedge \Theta^{(1,0)}$$

$$H = (\theta + \alpha_i) \wedge \omega_i \qquad [\omega_i] \in H^{1,1}(S)$$

$$dH = \omega_i \wedge \omega_i \neq 0$$

$$d\left(\|\Omega_X\| J \wedge J\right) = 0$$

$$N_1^2 + N_2^2 = c_2(T_S) - c_2(\mathcal{V}_S) = 24 - c_2(\mathcal{V}_S)$$

$$ds_X^2 \to t^2 ds_X^2$$

$$J_X \to t^2 J_X$$

$$H \to t^2 H \qquad \sim t^2 H N^2$$

$$R \wedge R \to t^0 R \wedge R \qquad \sim t^0 R \wedge R N^4$$

$$t^2 \sim N^2$$

$$R_\circ \sim t^{-2} N^2$$
$$\sim \mathcal{O}(1)$$

$$\begin{array}{ccc} S^1 & \to & S^3 \\ & & \downarrow \\ & & \mathbb{P}^1 \end{array}$$

$$ds_X^2 = e^u ds_S^2 + \theta \wedge \theta$$

$$\begin{array}{ccccc} T^2 & \xrightarrow[{[\omega]}]{} & X & & \\ & & \downarrow & & \\ & & K3 & \xleftarrow[J]{} & \nu \end{array}$$

$$\begin{array}{ccc} T^2 & \xrightarrow[\omega]{} & X \\ & & \downarrow \\ & & \mathbb{P}^2 \end{array}$$

$u(r)$
ODE

$\mathbb{C}^2/\mathbb{Z}_2$
$EH \leftarrow \nu = \emptyset.$

GLSM

2d gauge theory

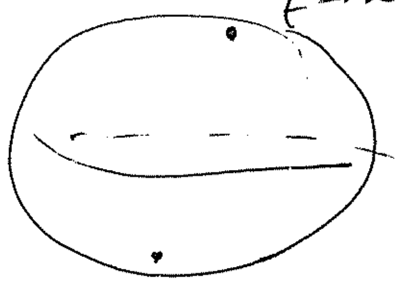
$\mathcal{M}_H = \mathcal{M}$

$F_a = [\phi_i, \phi_j] \longleftrightarrow H_a \in H^2(\mathcal{M})$

$D\phi = 0$

$\partial_{\mathcal{M}} = t^a H_a$

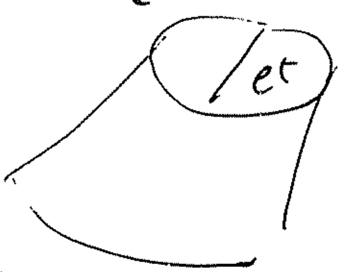
$t \to \infty$ Large radius CY



t^a

$t \to -\infty$ LG, exact orbifold

$T^2 \to X$
$\downarrow$



e^t

$\tilde{A}_1$

$\downarrow t \to \infty$

$T^2 \to$

$\mathbb{C}^2/\mathbb{Z}_2$

$(\mathbb{C}^2 \times T^2)/\mathbb{Z}_2$

2d gauge theory

(ϕ, ψ_+) (λ_-)

Q_i $\quad$ q_m

$$\sum_+ Q_i^2 - \sum_- q_m^2 = \mathcal{A} = c_2(T_{K3}) - c_2(V)$$

$$K3 \leftarrow V$$

Θ 

$$\mathcal{L}_\theta = (\partial\theta + NA)^2 + N\theta F \xrightarrow{\alpha} \mathcal{L}_\theta + N^2 \alpha F$$

$N^2 = \mathcal{A}$.

$$\left(\underset{\lambda_-}{T^2 \times \mathbb{C}^2}\right) / \mathbb{Z}_2$$

$$Z = \sum_{g h} h \square_g$$

$$Z = \sum_{g,h} e^{\mathcal{A}\int \alpha F} h \square_g$$

$\int F = \frac{g}{2}$ $\quad$ $\alpha = \frac{2\pi i}{2} h$ $\quad$ $\sum_{g,h} e^{\frac{\mathcal{A} g h \pi i}{2}} h \square_g$

$T^2 \times \mathbb{C}^2/\mathbb{Z}_2, \, \mathcal{V}_1$ $\qquad$ $T^2 \to \mathbb{C}^2/\mathbb{Z}_2, \, \mathcal{V}_2$

$\mathcal{A} = 0.$ $\qquad$ $\mathcal{A} = 2$

$H = 0, \, dJ = 0$ $\qquad$ $H \neq 0$, non Kähler

$\times \, \mathcal{M}_{T_2}$

$\times$ self-dual radius

at the self-dual point:

$\theta \leftarrow$ gauge invariant $\mathcal{A}_\theta = 0$ $\qquad$ $\mathcal{A}_\theta = 2$

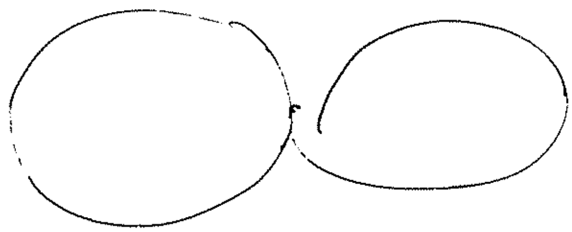
$\lambda_- \leftrightarrow \mathcal{A}_{\lambda_-} = 2$ $\qquad$ $\mathcal{A}_{\lambda_-} = 0$

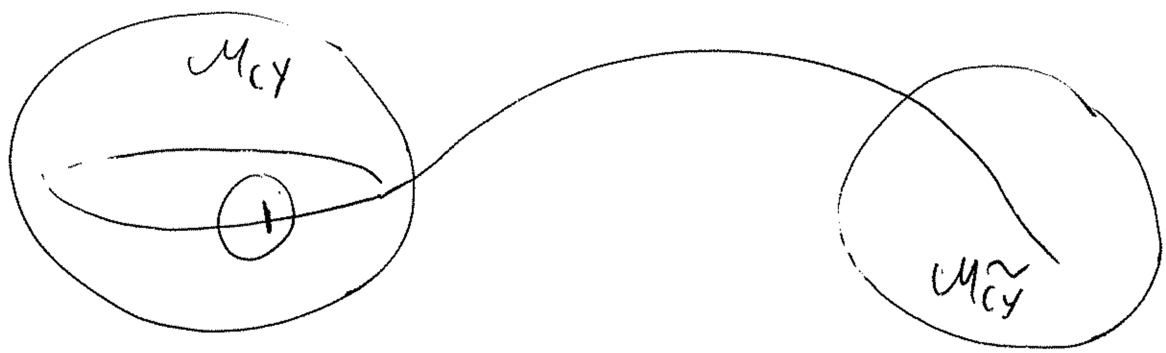
$t \to -\infty$ (vol $\mathbb{P}^1$) $\qquad$ $t \to -\infty$ (vol S^3)

CFTs are iso

$\lambda_- \to e^{H}$ ~~fermionize~~ bosonize

$e^{\theta} \to \chi_-$ fermionize





$$\begin{array}{lcc} & \phi_1 & \phi_2 \\ U(1) & 1 & -1 \end{array}$$

$$|(\partial + A)\phi_1|^2 + |(\partial - A)\phi_2|^2 + \left|\,|\phi_1|^2 - |\phi_2|^2 - t\right|^2 \quad (\mathrm{Re}(t) >> +1)$$

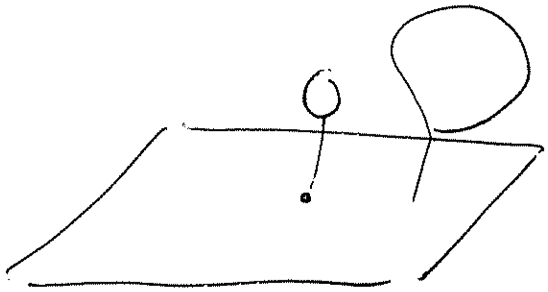
$t >> 1 \quad |\phi_1| >> 1 \Rightarrow A$ massive, ϕ_2 free $\mathbb{C}$.

$t << 1 \quad \ldots$

$$\begin{array}{ccc} \phi_1 & \phi_2 & P \\ +1 & +1 & -2 \end{array} \qquad \left(|\phi_1|^2 + |\phi_2|^2 - 2|P|^2 - r\right)^2$$

$$|\phi_1|^2 + |\phi_2|^2 = r \qquad S^3/U(1) = \mathbb{P}^1_r$$

$r >> +1$



$\langle P \rangle$

$$A(\phi) = \frac{\bar{\Phi}\,\partial\phi}{\Sigma|\phi|^2}$$

$$\mathcal{L} = \sum_i |(\partial + A(\phi))\phi|^2$$

$$g_{ij}(\phi)\,\partial\phi^i\partial\phi^j$$

EH $\sim \tilde{A}_{er}$

$r << -1$ $\quad P \neq 0 \quad U(1) \searrow \mathbb{Z}_2$

$$(\phi_1, \phi_2) \sim \mathbb{C}^2/\mathbb{Z}_2$$