

heterotic / type I string compactified

(2)

on a torus

$$T^n = \mathbb{R}^n / \mathbb{Z}^n$$

- flat metric on T^n
- 2-form (B-field)
- gauge bundle

$$G = \text{IOD gauge grp} \\ = (E_8 \times E_8) \rtimes \mathbb{Z}_2 \\ \text{or } \text{Spin}(32) / \mathbb{Z}_2 \quad (\text{het})$$

$$\text{SO}(32) \quad (\text{type I})$$

specified by

g_1, \dots, g_n gen's of $H_1(Y)$

$g_i \mapsto$ holonomy in gauge bdl $g_i \in G$

$$T(\text{Effective } G) = \mathfrak{u}(1) \oplus 2n$$

$\oplus T(\text{commutant of } g_1, \dots, g_n)$

for generic metric

typically $\mathfrak{u}(1) \oplus 16$
(generic pt of identity component of commuting n -tuples)

ADE singularities on K3

③

heterotic on
 T^4 / \mathbb{Z}^4

← dual →

type IIA on
K3

moduli space
in both cases

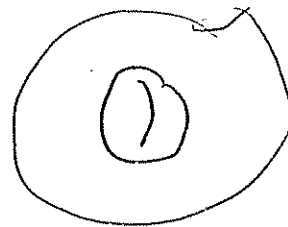
$$\left(\mathbb{R}^+ \times \mathcal{O}(4, 20) \right) / \mathcal{O}(4) \times \mathcal{O}(20)$$



expect non-abelian
gauge theories / sym
at certain param.
values

↔

2
Witten
(Schwarz,
Aspinwall)



$\mathbb{C}P^1 \rightarrow$ zero
area

explanation:

$$\left(\mathbb{R}^+ \times \mathcal{O}(4, 20) \right) / \mathcal{O}(4) \times \mathcal{O}(20) \quad \setminus \quad \mathbb{Z}^4$$

$\mathbb{C}P^1 \rightarrow$ zero
area

↔

param. approach
 \mathbb{Z}^4

K3 w/
ADE - sing

One can wrap a D2-brane around $\mathbb{C}P^1$; giving massive particle in effective theory mass \propto area ($\mathbb{C}P^1$)

At \bar{z} , get new massless particles; spectrum of new particles exactly corresponding to off-diag terms in gauge fields at non-abelian pts

($\bar{z} = \{ \text{param} \mid \text{eff gauge gap is non-abelian} \}$)

ADE singularities \leftrightarrow gauge sym. enhanced

related cases:

heterotic on $T^3 \longleftrightarrow$ M-theory on \mathcal{B}

$\mathbb{R}^+ \times (\mathcal{O}(3, 19) / \mathcal{O}(3) \times \mathcal{O}(19)) \bar{z}$




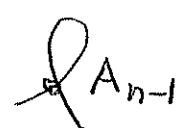


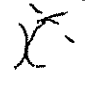

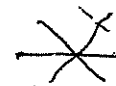

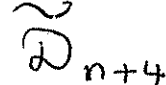
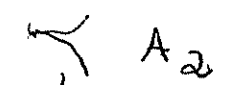
coupling \longleftrightarrow volume
M2-brane

heterotic on $T^2 \leftrightarrow$ "F-theory on $K3$ "

II B on base of an elliptic fibration (geometric data (j-invs) to build II B, varying coupling const, D7-branes at singular fibers)

Kodaira

local singularities in 1-param. ell fibrations (w/ section)

<u>type</u>	<u>monodromy on $H_1(T^2)$</u>	<u>n/s geomety</u>	<u>Weierstrass</u>
I_0	id		
I_n	$\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$	 sect.	
II	order 2		
III	order 4		
IV	order 3		
I_n^*	$\begin{pmatrix} -1 & n \\ 0 & -1 \end{pmatrix}$	 D_{n+4}	 D_{n+4}
\downarrow			

II*	6	\tilde{E}_8	E_8
III*	4	\tilde{E}_7	E_7
IV*	3	\tilde{E}_6	E_6

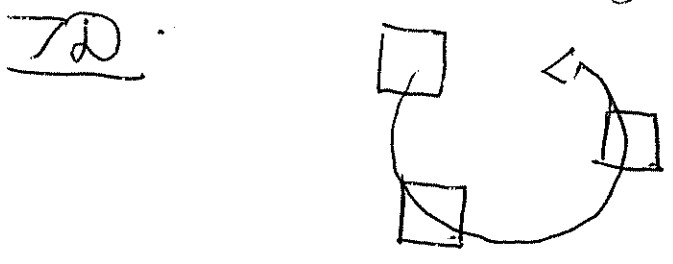
(not one-to-one correspondence
you have A_1, A_2 appearing twice)
 $y^2 = Wx^3 - g_4(s)x - g_6(s)$

(F-theory on B) $\times S^1 \iff$ M-theory
on total space
of ell. fibr

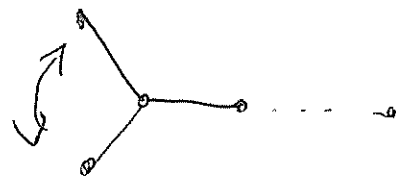
singularity in
W model determining
non abelian gauge group

- I_n : n -D7-branes
- I_n^* : n D7-branes + orientifold pbr

8D : ADE gauge groups (F-theory)



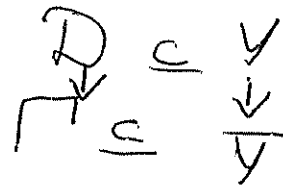
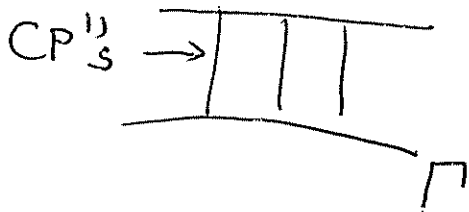
monodromy on
ADE group around S^1
 \rightarrow gauge group w/
non-simply laced alg.



6D: F-theory (5D M-theory
4D II A)

~~K3~~ T^2

CY-3 fold



spectrum of
 D2/M2-brane

on 1-gluon-param.

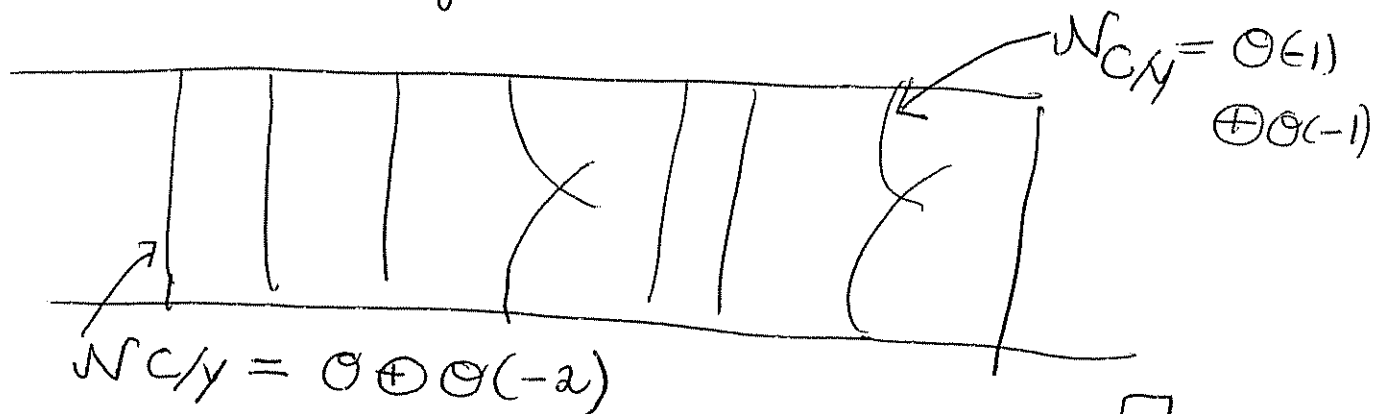
family of CP^1 's

= (if no monodromy, no sing fibres)

(g-1) adjoint hypermult +
 gauge mult.

* CP^1_y can degenerate at special values $y_0 \in \Gamma$

(8)



* The components of special fibers are rigid (unless two such divisors are colliding)

* $D2/M2$ - branes wrapping n periods

= $(\frac{1}{2})$ hypermultiplet in spectrum

* Gauge charges are given by

$$D \cdot C \neq 0 \text{ for } C \subseteq D$$

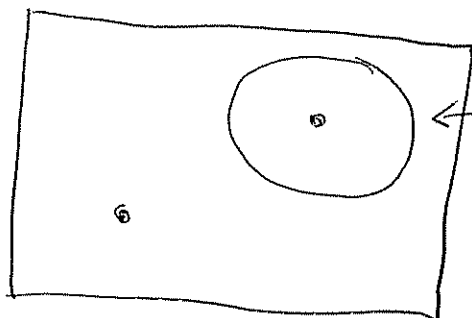
Γ $xy = z^2 + wt^2 + t^3$

Another application

9

II B models, $\dim_{\mathbb{C}} Y = 3$
and we have spacetime-filling
D3-branes

$p \in Y$ smooth pt

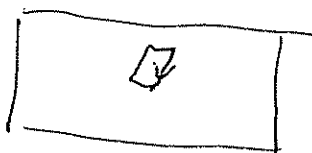


Ads/CFT
approach to
brane

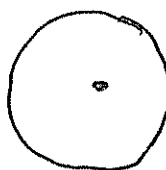
$S^5 =$ boundary of
ball near pt

$S^5 \times AdS_5$

$p \in Y$ is a sing pt



D5 or D7
branes
wrapping
cycles in
resolution



$M^5 =$ body of a
small nbhd

$M^5 \times AdS_5$ models
physics

\leadsto model physics
(quiver rep)

- works well in some cases \checkmark
- works awkwardly in more
- not know in general