Morrison: Much ado about $N=2$

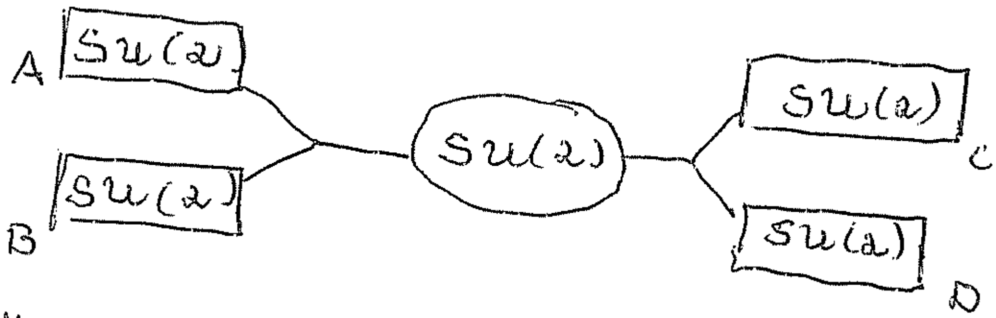
Gaiotto 0904.2715
"$N=2$ dualities"

$N=2$ conformal field theories in 4D
$\rightarrow$ Seiberg-Witten curves (Riemann-surfaces)

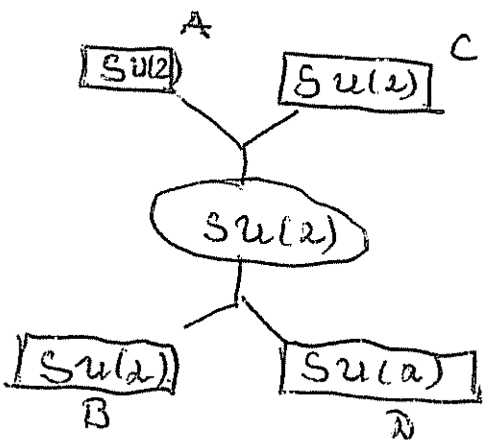
$\rightarrow$ a punctured Riemann surface

$SU(2)$ w/ $N_f = 4$; flavor symmetry $SU(4)$, enhanced to $SO(8)$
$SO(8) \supset SO(4) \times SO(4)$

locally $SO(4) = SU(2) \times SU(2)$

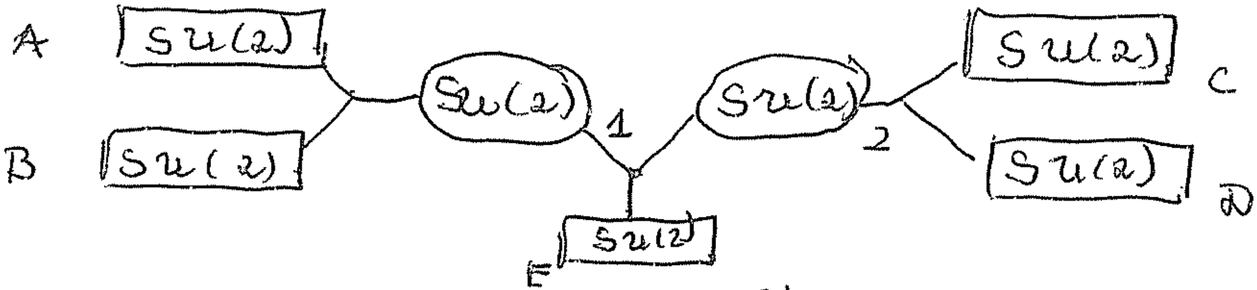


claim: different weakly coupled limits
$\longleftrightarrow$ different groupings of $\{A, B, C, D\}$ in pairs



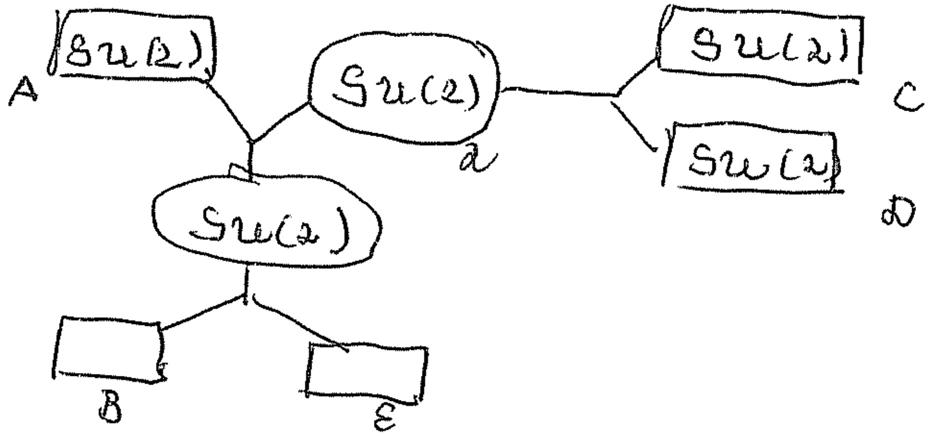
or the other

linear quiver, n $SU(2)$'s,
bifundamental for adjacent,
2 hypers at cubic



flavor sym of bifundamental

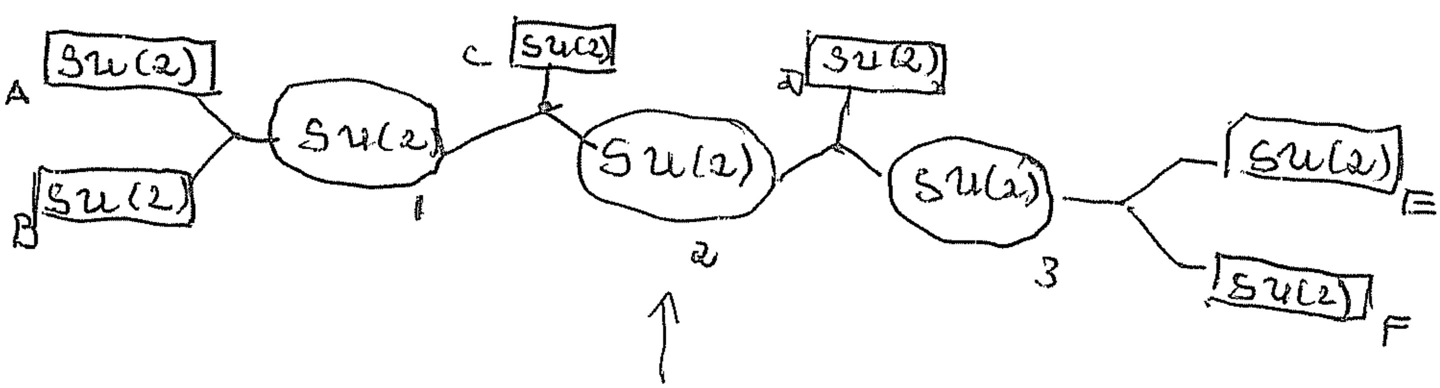
if $SU(2)_2$ decoupling

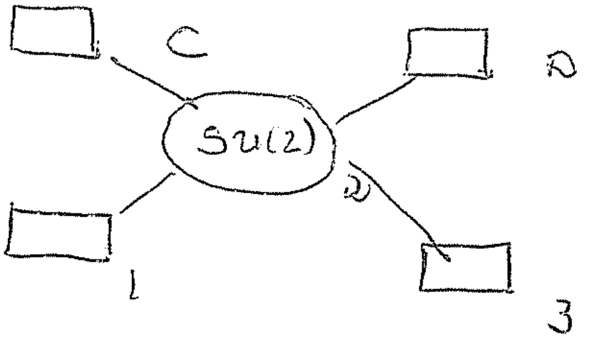


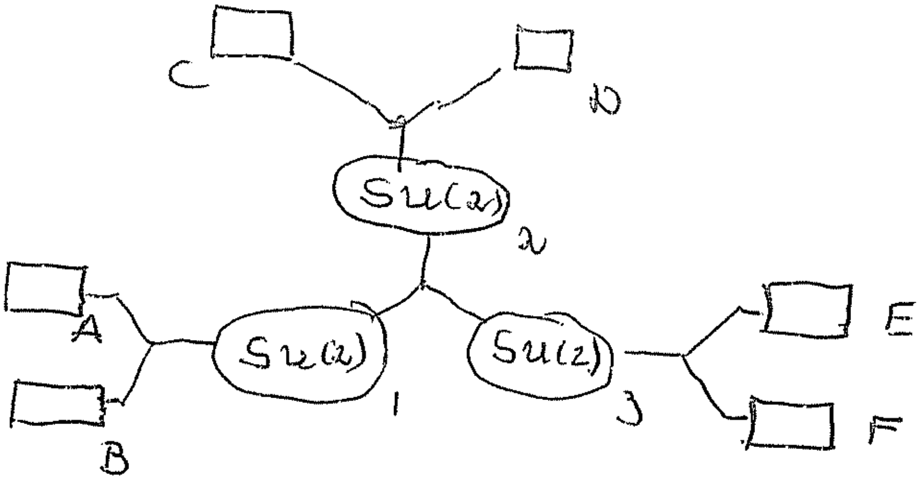
S-duality permutations of $SU(2)_1$, $SU(2)_2$ do not commute

... subgroup of S_5

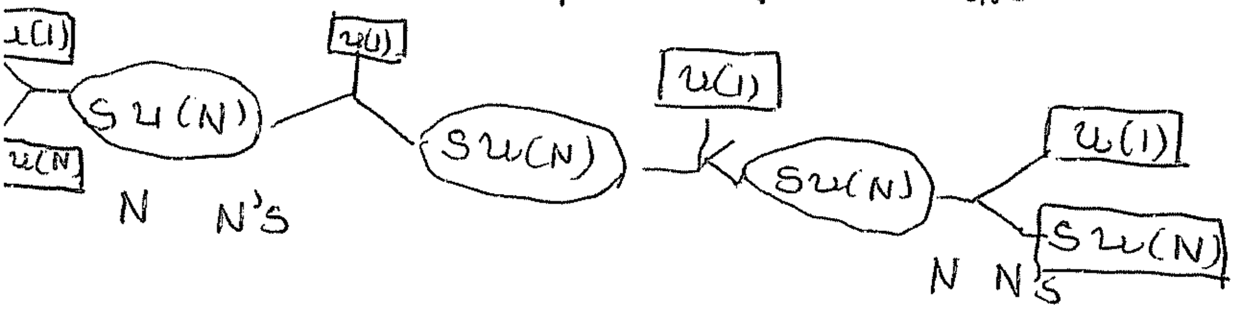
$n=3$







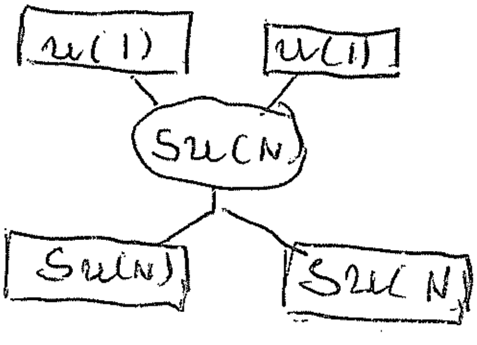
same linear quiver for $SU(N)$



$SU(N) \quad N_f = 2N, \quad N > 2$

$SU(2N) = U(N) \times U(N)$

limit:



tricky to describe

$SU(2) \quad \mathcal{H}_y / SL(2;\mathbb{Z})$

$SU(N), \; N > 2 \quad \mathcal{H}_y / \Gamma_0(2)$

$\Longleftrightarrow$ extra cusp

<u>$N=3$</u> : Gaiotto's claim:

at weak coupling w.r.t new cusp

gauge sym is $SU(2) \subseteq SU(3)$

$SU(2) \subseteq E_6$ global sym

$SU(3)$ . . . $SU(2)$ in $SU(3)$ — $\boxed{u(1)}$

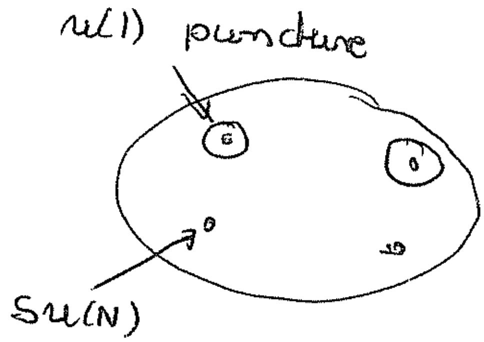
E_6 has 1-dim moduli space

$\mathrm{Tr}\,\phi^3 \longleftrightarrow \quad \longleftrightarrow \ldots$ < dim 3 operator>

dim 2 operator in $SU(3)$ $\longleftrightarrow$ . . . . dim 2 operator $SU(2)$

commutant of $SU(2)$ in $SU(6)$

$SU(6) \times u(1)$



Seiberg-Witten curves

$\boxed{N} - (N) - (N) - (N) - \boxed{N}$

D4 / NS 5 brane rep

lifts to M-theory

Witten proposed a SW-curve

$\underline{N=2}$ n $SU(2)$'s

t, v

$$v^2 t^{n+1} + c_1 (v^2 - u_1) t^n + \dots + c_n (v^2 - u_n) t + c_{n+1} v^2 = 0$$

where $u_1, \dots, u_n$ describe Coulomb branch

$u_i \sim \mathrm{Tr}\, \phi_n^2$ at weak coupling

$c_1, \dots, c_n \longleftrightarrow$ gauge couplings.

$$v^2 (t^{n+1} + \dots + c_{n+1}) = u_1 t^n + \dots + u_n t$$

$$LHS = v^2 \prod_{a=0}^{n} (t - t_a)$$

$$\tau_a = \frac{1}{\pi i} \log\left(\frac{t_{a-1}}{t_a}\right)$$

$v \log t_0 = 1$

SW differential $\lambda = v \dfrac{dt}{t}$

$\underline{n=1}$

$v^2(t-1)(t-t_1) = ut$

$$\lambda = \frac{\sqrt{u}}{\sqrt{t(t-1)(t-t_1)}}\, dt$$

$\tau \longleftrightarrow y^2 = t(t-1)(t-t_1)$

$\tau \to i\infty$ permutations in pts
$\longleftrightarrow$ S-duality

$$t \to \frac{az+b}{cz+d}, \quad x \to (cx+d)^2 x$$

$x^2 = \frac{u}{\Delta_4(z)}$ t_1 = cross-ratio of roots

$\lambda = x\, dz$

$x^2 = \varphi_2(z)$ z = coord in punctured Riem. surface Σ

$\varphi_2(z)\, dz$ is a quadratic differential on Σ

$x\, dz$ SW diff.

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z = coord on Σ
x = — " — on $T^*\Sigma$

moduli = {$(n+3)$- punctured sphere}
= $\mathcal{M}_{\dots}$

Mass deformation

$$v^2 \prod_0^n (t - t_a) = v\, M_{n+1}(t) + \underbrace{u_{n+1}(t)}_{\substack{t^{n+1},\ t^0 \text{ term} \\ \text{new}}}$$

$n+3$ parameters

SW diff has poles
(Residues given in terms of mass parameters)

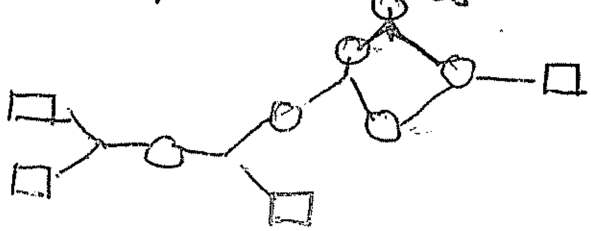
$$\leadsto \quad v^2 \prod (t - t_a)^2 = u_{n+1}(t) \prod (t - t_a) - \frac{1}{4} M_{n+1}^2 (t)$$

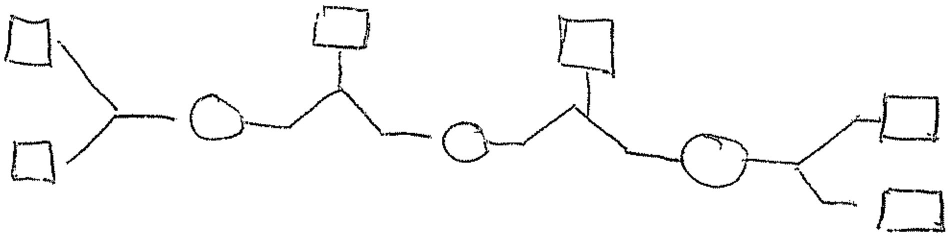
λ can be shifted, make unessential change

$$\lambda = v \frac{dt}{t}$$

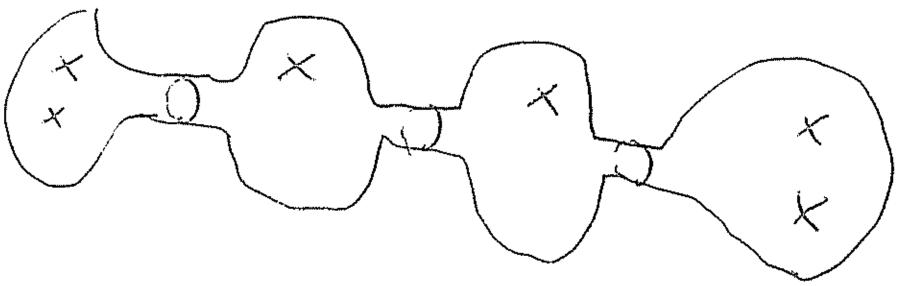
at each puncture, 2 inv images have poles, opposite residues

g loops in quiver
n punctures





RS:



another weakly coupled limit: