M5-branes wrapped on Riemann surfaces and anomalies

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Plan

- Motivations – results
- N=2 theories from wrapped M5-branes
- N=1 theories
- Gravity dual
- Anomaly polynomial and central charges
- Conclusions – future directions

Based on:	FB, Benvenuti, Tachikawa	0906.0359
	FB, Tachikawa, Wecht	0909.1327
	Alday, FB, Tachikawa	0909.4776

Motivations

- understanding the M5-brane theory
  - 1 M5: self-dual $B_{\mu\nu}$, $\phi^{i=1\ldots5}$, $\psi^{a=1,2}$
  - N M5: 6d N=(2,0) SCFT with $SO(5)_R$
- S-duality
- new isolated SCFT *without* Lagrangian
- BPS quantities exactly computable via 2d - 4d correspondence with Liouville/Toda

M5-branes on Σ with N=2

- N M5-branes wrapped on a Riemann surface $\Sigma_{g,n}$ with n punctures, with N=2 twist (→ the normal bundle is $T^*\Sigma$)

  → N=2 SCFTs whose diagram "reproduces" the surface Σ

Gaiotto

- A Riemann surface with punctures admits pant decompositions, *e.g.*:

  $3\,(g-1)+n$ tubes
  $2\,(g-1)$ triskelions completely glued
  n triskelions with one free puncture

- For each decomposition, consider a limit (for the complex structure) with very long tubes

- Long tube → weakly coupled gauge group
  The pant is *not* weakly coupled: isolated SCFT

M5-branes on Σ with N=2

- The complex structure moduli space of $\Sigma_{g,n}$ is

$$M_{g,n} = \widetilde{M}_{g,n}/\Gamma \qquad\qquad \Gamma = \pi_1(M_{g,n})$$

Teichmüller/(large diffeo x permutations)

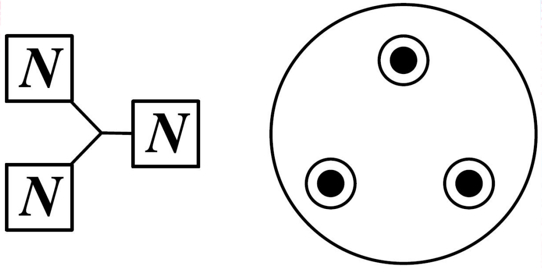
It equals the parameter space of marginal couplings of the 4d theory (Γ being S-self-duality)

- Different pant decompositions *give* S-dual descriptions

- Particularly simple class of theories:

  - T$_g$: no punctures ↔ no flavor symmetry
  - T$_{g,n}$: only "maximal" punctures ↔ $\mathrm{SU}(N)^{\#}$ flavor symm
  - T_N: sphere with 3 "maximal" punctures

T_N theory



- Theory on N M5-branes wrapped on S^2 with 3 "maximal" punctures

- N=2 isolated SCFT, with $\mathrm{SU}(N)^3$ flavor symmetry

- Coulomb branch parametrized by dimension-k operators

$$u_k^{(i)} \quad \text{for} \quad k = 3 \ldots N, \quad i = 1 \ldots k-2 \qquad \text{flavor singlets}$$

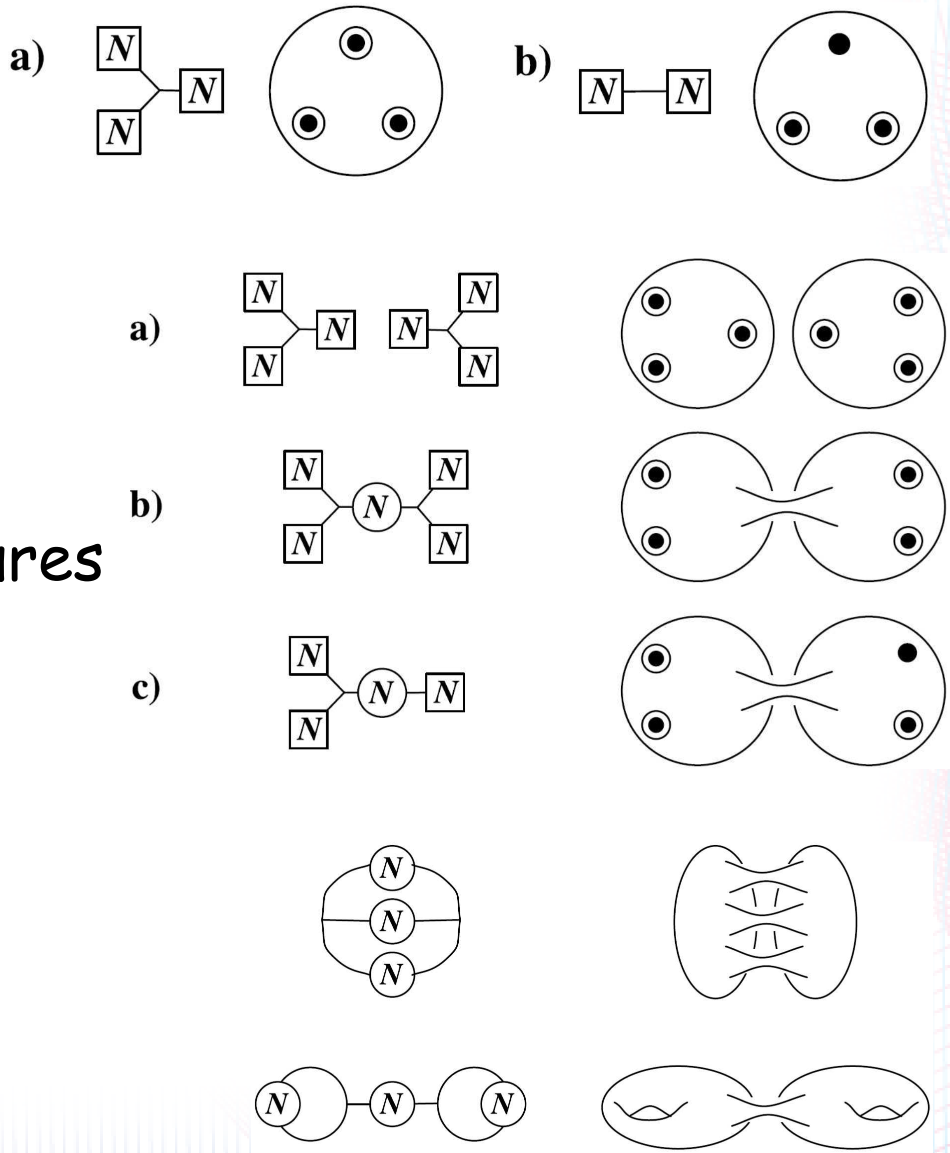
- Higgs branch (?) contains dimension-2 operators

$$\mu_{i=1,2,3} \qquad \text{adjoint of } \mathrm{SU}(N)_i \text{ flavor}$$

- Examples: T_2 – 8 free $\frac{1}{2}$hypers $Q_{\alpha\beta\gamma}$
  T_3 – E_6 theory of Minahan-Nemeshansky

Sicilian theories

- Building blocks:
  a) T_N theory
  b) free hypermultiplets
  c) generic triskelions...
- Glue (gauge) maximal punctures together (→ SCFT)
- Generate *Sicilian* theories:
  a) $T_{g,n}$
  b) many more...



The Coulomb branch

- N=2 supersymmetry: $T^*\Sigma$ hyperKahler (½ SUSY) with M5-branes wrapping a holomorphic curve (½ SUSY) of degree N:

$$x^N = \phi_N(z) + \phi_{N-1}(z)\, x + \ldots + \phi_2(z)\, x^{N-2}$$

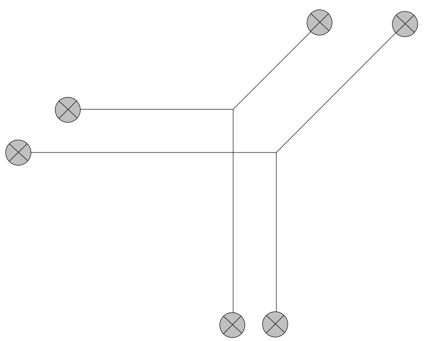
- Mass of M2-branes: SW differential $\lambda_{SW} = x\, dz$
- Mass of BPS particles: $m = \oint \lambda_{SW} = \oint x\, dz$
massless/massive hypers: $\phi_j(z)$ degree up to j-1 or j
- Coulomb branch: moduli space of multi-differentials with allowed poles. Riemann-Roch:

$$\text{moduli of } \phi_j = (2j-1)(g-1) + \sum_{p=1}^{n} d_{p,j}$$

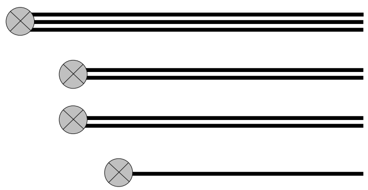
Flavor symmetry at the puncture

- Understand classification of pole structures and associated flavor symmetry from IIB construction.
  Focus on T_N theory:

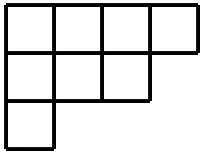
  (p,q) 5-brane webs compactified on S^1 realize the T_N theory



- Generalization: end multiple 5-branes on the same 7-brane



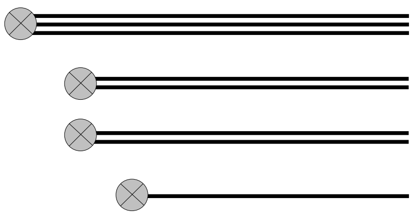
- Punctures classified by *partitions of* N
  $\leftrightarrow$ Young tableaux with N boxes

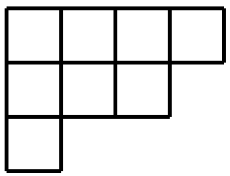


Flavor symmetry at the puncture

- Each stack of n_k identical objects
  → $U(n_k)$ symmetry:

$$S\left[\prod_{k\geq 1} U(n_k)\right]$$





- Non-maxiamal punctures as effective theories along the Higgs branch of the maximal puncture

M5-branes on Σ with N=1

- Worldvolume point of view: wrap the branes with N=1 twist

$$\begin{array}{ccc} \mathrm{SO}(5)_R & \rightarrow & \mathrm{SO}(4) \simeq \mathrm{SU}(2)\times\mathrm{SU}(2)_F \\ & & \downarrow \\ & & \mathrm{U}(1)_R \end{array}$$

embed the spin connection $\mathrm{U}(1)_\Sigma$ into $\mathrm{U}(1)_R$

- We expect a new infinite family of N=1 SCFTs, with intricate net of S-dualities

- We will see what characterizes their moduli

Massive deformed $T_{n,g}$

- Field theory point of view: deform N=2 to N=1 with mass for the adjoint scalars

- Closed class under S-duality:
  only gauge groups provide dimension-2 operators $\mathrm{Tr}\,\Phi_s^{\,2}$

- N=2 SUSY requires $$W=\sum_s \mathrm{Tr}\,\Phi_s(\mu_{a,i}+\mu_{b,j})$$
- Mass deformation $$\delta W=\sum_s m_s \mathrm{Tr}\,\Phi_s^2$$
- The theory flows to a fixed point with "quartic" superpotential

$$\rightarrow \qquad W=\sum_s \frac{1}{m_s}\mathrm{Tr}(\mu_{a,i}+\mu_{b,j})^2$$

NSVZ formula

- The all-loop beta-function is computed by the NSVZ formula It depends on the representation and anomalous dimension of fundamental fields.

→ what with non-Lagrangian sector?

$$\beta_{\frac{8\pi^2}{g^2}} = 3\,\mathrm{T}[\boldsymbol{adj}] - \sum_i \mathrm{T}[\boldsymbol{r}_i](1-\gamma_i) + K$$

$$K\,\delta^{ab} = 3\,\mathrm{Tr}\,R_{N=1}\,T^a\,T^b$$

← 't Hooft anomaly

- The low-energy N=1 R-symmetry is a combination of $\mathrm{U}(1)_R \times \mathrm{SU}(2)_R$

$$R_{IR} = \frac{1}{2}R_{N=2} + I_3$$

Flavor current central charge $\quad k_G\,\delta^{ab} = -2\,\mathrm{Tr}\,R_{N=2}\,T^a\,T^b$

T_N and sons: $\quad k_G = 2N$

Conformal manifold

- Compute the dimension of the conformal manifold (exactly marginal deformations) à la Leigh-Strassler:
  - write down all marginal operators, *e.g.* $\mathrm{Tr}\,\mu^2$, $\mathrm{Tr}\,\mu\mu$, $\mathrm{Tr}\,W_\alpha W^\alpha$
  - count real relations from vanishing beta-functions
  - count phases removed by field redefinitions
- No punctures: $\dim_{\mathbb{C}} M_C = 6(g-1)$

  Maximal punctures: $\dim_{\mathbb{C}} M_C = 6(g-1) + 2n_N$

- **Central charges**:

$$a = (g-1)\,\frac{9N^3 - 3N - 6}{32} \qquad c = (g-1)\,\frac{9N^3 - 5N - 4}{32}$$

Gravity dual

- Maldacena-Nunez solution with N=1 twist:

$$w[AdS_5] \times w[H^2] \times \tilde{S}^3 \times I$$

squashed S^3 preserves $\mathrm{U}(1)_R \times \mathrm{SU}(2)_F$, fibered over H^2

- To get compact Riemann surface, mod out by Fuchsian group:

$$C = H^2/\Gamma \qquad \Gamma \subset \mathrm{SL}(2,\mathbb{R})$$

- $\mathrm{SU}(2)_F$ "unwanted" symmetry
Introduce $\mathrm{SU}(2)_F$ Wilson lines (N=1) on C:

$$\tilde{S}^3 \to E \to H^2 \qquad E_C = E/\Gamma_W \qquad \Gamma_W \subset \mathrm{SL}(2,\mathbb{R}) \times \mathrm{SU}(2)_F$$

- Moduli space of Σ_g with $\mathrm{SU}(2)$ Wilson lines: $6(g - 1) = \dim M_C$

Gravity dual with maximal punctures

- The gravity dual (Gaiotto Maldacena) of the maximal puncture is a Z_N orbifold singularity

$$(z, x_1, x_2) \to (e^{2\pi i/N} z, e^{-2\pi i/N} x_1, x_2)$$

- The N=1 twist alone would give the action

$$(z, x_1, x_2) \to (e^{2\pi i/N} z, e^{-\pi i/N} x_1, e^{-\pi i/N} x_2)$$

→ the $\mathrm{SU}(2)_F$ monodromy has fixed conjugacy class:

$$(x_1, x_2) \to (e^{-2\pi i/N} x_1, e^{2\pi i/N} x_2)$$

- Moduli space of $\Sigma_{g,n}$ with $\mathrm{SU}(2)$ Wilson lines and constrained monodromies: $6(g - 1) + 2n_N = \dim M_C$

Central charges from SUGRA

- Central charges computed by AdS_5 radius and curvature corrections

- Packaged in the anomaly polynomial of the 6d theory:

$$I_8[A_{N-1}] = \frac{N-1}{48}\left[p_2(\mathrm{N}) - p_2(\mathrm{T}) + \frac{1}{4}\left(p_1(\mathrm{N}) - p_1(\mathrm{T})\right)^2\right] + \frac{N^3 - N}{24} p_2(\mathrm{N})$$

Witten Harvey Minasian Moore

- Chern roots: tangent bundle $\pm\lambda_1$, $\pm\lambda_2$, $\pm t$ – normal bundle $\pm n_1$, $\pm n_2$
Highlight the $\mathrm{U}(1)_R$ bundle, impose N=1 SUSY and integrate on C

$$n_{1,2} \to n_{1,2} + c_1(F) \qquad n_1 + n_2 + t = 0 \qquad \int_C t = 2 - 2g$$

- Compare with the anomaly polynomial of the 4d theory:

$$I_6 = \frac{\mathrm{Tr}\, R^3}{6} c_1(F)^3 + \frac{\mathrm{Tr}\, R}{24} c_1(F)\, p_1(\mathrm{T}_4)$$

Central charges from SUGRA

- Get: $\mathrm{Tr}\, R = N\,(g - 1)$, $\mathrm{Tr}\, R^3 = N^3\,(g - 1)$

- Exploiting SUSY, reproduce the central charges:

$$a = \frac{3}{32}\left[3\,\mathrm{Tr}\,R^3 - \mathrm{Tr}\,R\right] \qquad c = \frac{1}{32}\left[9\,\mathrm{Tr}\,R^3 - 5\,\mathrm{Tr}\,R\right]$$

→ matching with field theory

2d – 4d correspondence

- The Nekrasov partition function of the 4d theory obtained wrapping M5s on $\Sigma_{g,n}$ with equivariant deformations ε_1, ε_2 is equal to the conformal blocks of Liouville/Toda theory on $\Sigma_{g,n}$ with parameter $b^2 = \varepsilon_1 / \varepsilon_2$

  Alday Gaiotto Tachikawa

- More observables: Wilson – 't Hooft loops, surface operators, …

  Drukker Morrison Okuda

  Alday Gaiotto Gukov Tachikawa Verlinde

  Drukker Gomis Okuda Teschner

- Classical equivalence: Coulomb branch is the space of multi-differentials = classical solutions of Liouville/Toda

  Bonelli Tanzini

- Quantum properties?

Toda central charge from 6d anomaly

- Compute the 2d central charge compactifying the anomaly polynomial
  Compactify the N = (2,0) theory on $\Sigma \times X_4$

- 6d anomaly polynomial for ADE series without center of mass:

$$I_8[G] = \frac{r_G}{48}\left[p_2(\mathrm{N}) - p_2(\mathrm{T}) + \frac{1}{4}\left(p_1(\mathrm{N}) - p_1(\mathrm{T})\right)^2\right] + \frac{d_G h_G}{24} p_2(\mathrm{N})$$

Intriligator; Yi

- The twist is more involved: 2 steps.

- First step: embed the spin connection of X_4 in $\mathrm{SO}(5)_R$:

$$\mathrm{SO}(5,1)\times\mathrm{SO}(5)_R \to \mathrm{SO}(1,1)\times\mathrm{SU}(2)_l\times\mathrm{SU}(2)_r\times\mathrm{SO}(2)_R\times\mathrm{SO}(3)$$

$$\mathrm{SU}(2)_r \to \mathrm{diag}\left[SU(2)_r\times SO(3)\right]$$

- Highlight the $\mathrm{U}(1)_R$ bundle, impose SUSY and integrate on X_4:

$$n_1 \to n_1 + 2c_1(F) \qquad n_2+\lambda_1+\lambda_2=0 \qquad \int_{X_4}\lambda_1\lambda_2=\chi(X_4) \qquad \int_{X_4}\lambda_1^2+\lambda_2^2=P_1(X_4)$$

- Compare with 2d anomaly polynomial, and use N=(0,2) SUSY:

$$I_4 = \frac{c_R}{6} c_1(F)^2 + \frac{c_L - c_R}{24} p_1(\mathrm{T}_2)$$

- We get the central charges of the 2d theory on Σ:

$$c_R = \frac{1}{2}(P_1 + 3\chi) r_G + (P_1 + 2\chi) d_G h_G$$

$$c_L = \chi r_G + (P_1 + 2\chi) d_G h_G$$

- Nekrasov's partition function is an equivariant integral on $X_4 = \mathbb{R}^4$ with equivariant parameters $\varepsilon_{1,2}$ with respect to a $U(1)^2$ action. Integrated equivariant classes (use localization formula):

$$P_1(\mathbb{R}^4) = \int \epsilon_1^2 + \epsilon_2^2 = \frac{\epsilon_1^2 + \epsilon_2^2}{\epsilon_1 \epsilon_2} \qquad \chi(\mathbb{R}^4) = \int \epsilon_1 \epsilon_2 = 1$$

- Second step: embed the spin connection of Σ into $U(1)_R$
→ topological twisting of right sector $\qquad b^2 = \varepsilon_1 / \varepsilon_2$

$$c_R \to 0 \qquad c_L = r_G + \left(b + \frac{1}{b}\right)^2 d_G h_G$$

Future directions

- Higgs branch
- more on N=1
  compactify the 6d N=(1,0) theory
- 2d – 4d correspondence: a proof
- surface operators, domain walls, …
- a gravity dual of large N Toda, or SUSY versions of it?