6 November 2009
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More about S-dual probes of supersymmetric gauge theory

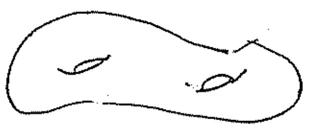
6-D ← M5-brane $N = \#M5$'s

"inspiration"

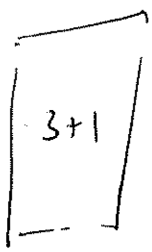
4-D ← $\mathcal{N}=2$ SUSY gauge theories   S-duality

Instanton + SW

(Nekrasov)



Σ



2-D   Liouville/Toda CFT

AGT

$T(z)$, $V_m(z)$,

conformal blocks

$$\mathcal{M}_{g,n} = T_{g,n}/\Gamma_{g,n}$$

6D → 2D

after dim. red.

⇒ Liouville/Toda CFT

Liouville ↔ $N=2$

Toda ↔ general N

SU(2)

On top of Σ: SW differential

λ_{SW} ← 1-form

$\lambda_{SW}^2 = \phi_2(z)$

Σ_{SW} = double cover of Σ $\qquad x^2 = \phi_2(z)$.

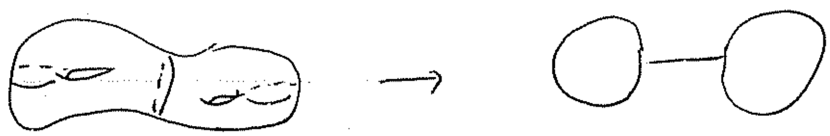
↑ because SU(2)

ϕ_2 is a quadratic differential

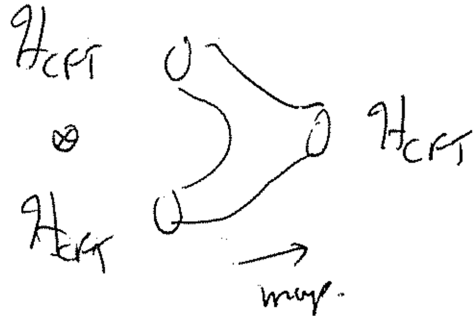
$= \sum u_i \, \phi_2^{(i)}(z)$ ↖ basis of quadratic differentials

↗ Coulomb branch parameters

Conformal blocks

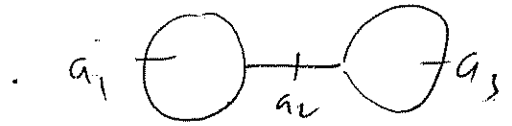


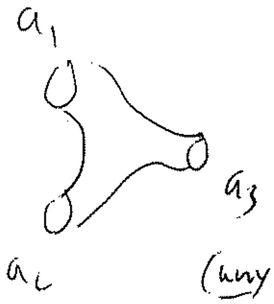
CFT Hilbert space each channel
(channel = dividing circle)



$$\mathcal{H}_{CFT} = \bigoplus_{\alpha} \mathcal{H}_{CFT}^{left}(\alpha) \otimes \mathcal{H}_{CFT}^{right}(\alpha) = \sum_a [a]_L \otimes [a]_R$$

Liouville momentum

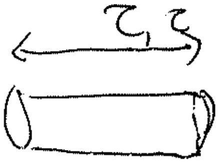




(any a_3 can come out)

$$Z = \int da_i \, |Z(a_i)|^2$$

conformal blocks



(length + twist)

$$e^{\tau L_0 + \bar\tau \bar L_0}$$

$(\tau_i, \bar\tau_i) \ldots$ $\quad Z(a_i)$ = holo function of moduli

curvature

$$S_{2D} = \int \partial\varphi \bar\partial\varphi + \mu e^{\varphi} + Q R \varphi.$$

$$Q = b + \tfrac{1}{b}$$

CFT with central charge $1 + 6Q^2 \geq 25.$

$$T = (\partial\varphi)^2 + Q \partial^2 \varphi$$

$$\{Q, \bar Q\} = U(1) \text{ generator}$$

Pestun on S^4

1-loop det

$$Z_{4\text{-}d} = \int da_i \, |Z(a_i)|^2$$

"Coulomb branch parameters"

2+1-dim

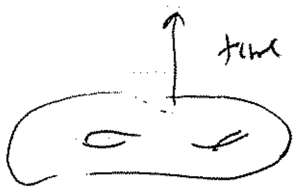
WZW → CS theory:

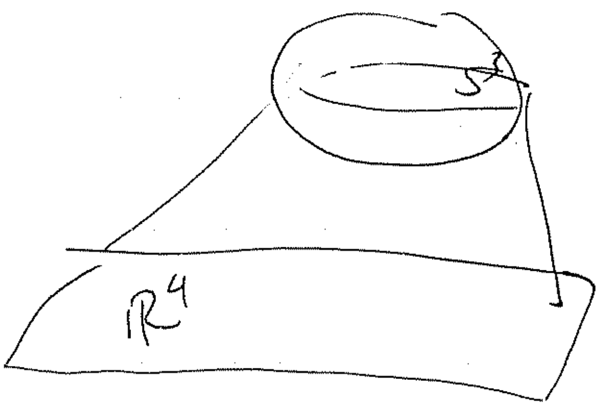
$$\psi(A) = \langle e^{AJ} \rangle_{2D\,CFT} \qquad \text{(left-movers)}$$

"SL(2,R) CS theory": $\psi(\mu) = \langle e^{\int \mu(z) T(z)} \rangle_{Liouville}$ (in a conformal block)

on Σ: $F(A) = dA + A \wedge A = 0$

$\longleftrightarrow$ constant curvature metrics (Teichmüller space $T_{g,n}$)





1-loop in Pestun $\longleftrightarrow$ DOZZ factor

$$W_j = \mathrm{tr}_j\, P\exp \oint (A + i\phi) \quad \longleftrightarrow \quad \mathrm{tr}_j\left(e^{ia\tau_3}\right) \overset{\chi_j(a)}{=}$$

$$\langle \phi \rangle = a$$

$$\chi_{j_1}(a)\,\chi_{j_2}(a) = \sum_{|j_1-j_2| \le j_3 \le (j_1+j_2)} \chi_{j_3}(a)$$

rep. ring of SU(2)

In Liouville Theory, have noncompact a

and SU(2) reps $\longleftrightarrow$ "degenerate fields"

$$V_a = e^{2\alpha\varphi}, \quad \alpha = \frac{Q}{2} + ia$$

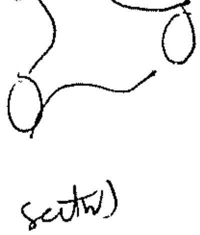
$Q = b + \frac{1}{b}$ $\qquad e^{\mu\varphi}$

$$e^{-\frac{bj}{2}\varphi} = \Phi_{j,1}$$

$$\Phi_{1,j} = e^{-\frac{1}{2b}j\varphi}$$

(at $b=1$, $\Phi_{1,j} = \Phi_{j,1}$)

j_1



j_2 $\qquad$ j_3, satisfying SU(2) rep. ring.

($e^{-\frac{bj}{2}\varphi}$ sector)

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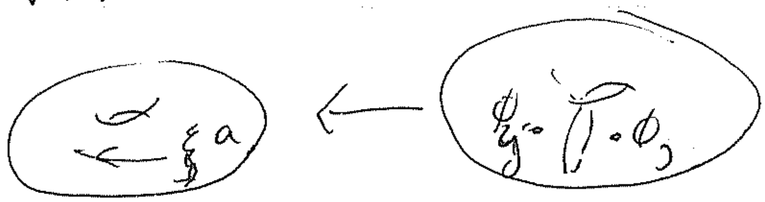
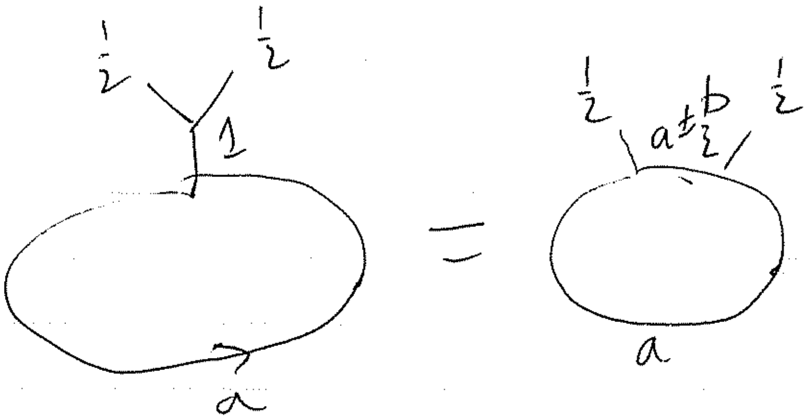
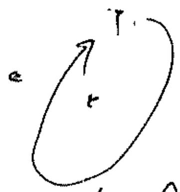


$\phi_j * \phi_j = 1 + \cdots$ chiral operators





phase factor

$$= \chi_{\frac{1}{2}}(a)$$

A-cycle operator $\phi_j(A)$

B-cycle operator: $a \to a \pm b$

$\phi_j(B)$:

$$\phi_j(A)\, Z(a) = \chi_j(a)\, Z(a)$$
$$\phi_j(B)\, Z(a) = Z(a + \tfrac{b}{2}) + Z(a - \tfrac{b}{2})$$

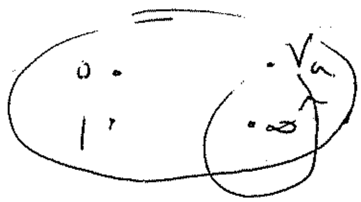
(rep. of the fusion algebra)

$Z_A(a_1) = S_{a_1 a_2} Z_B(a_2)$ "S diagonalizes the fusion rules"

$\tau = \theta + \frac{i}{g_{YM}^2} \longrightarrow -1/\tau$ = magnetic dual coupling

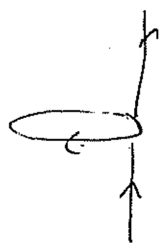
$\phi(A) \longleftrightarrow$ Wilson lines

$\phi(B) \longleftrightarrow$ "'t Hooft lines"

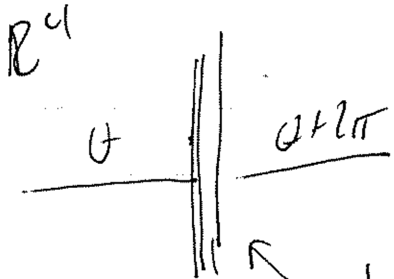


$\tau = \theta + \frac{i}{g_{YM}^2}$ $\theta \to \theta + 2\pi$

pair creation!



domain wall in theory



induced CS field theory on domain wall

$\int A \wedge dA + \frac{2}{3} A^3$