Morrison: K3 surfaces, modular forms and non-geom compactification

	ord f	ord g	ord Δ	sing	monodromy
I_0	≥ 0	≥ 0	0	–	$\begin{pmatrix}1&0\\0&1\end{pmatrix}$
I_n	0	0	n	A_{n-1}	$\begin{pmatrix}1&n\\0&1\end{pmatrix}$
II	≥ 1	1	2	–	$\begin{pmatrix}1&1\\-1&0\end{pmatrix}$
III	1	≥ 2	3	A_2	$\begin{pmatrix}0&1\\-1&0\end{pmatrix}$
IV	≥ 2	2	4	A_3	$\begin{pmatrix}0&1\\-1&-1\end{pmatrix}$
I_0^*	≥ 2	≥ 3	6	D_4	$\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$
I_n^*	2	3	$n+6$	D_{n-4}	$\begin{pmatrix}-1&n\\0&-1\end{pmatrix}$
IV^*	≥ 3	4	8	E_6	$\begin{pmatrix}-1&-1\\1&0\end{pmatrix}$
III^*	3	≥ 5	9	E_7	$\begin{pmatrix}0&-1\\1&0\end{pmatrix}$
II^*	≥ 4	5	10	E_8	$\begin{pmatrix}0&-1\\1&1\end{pmatrix}$
non-minimal	≥ 4	≥ 6	≥ 12		

Physical theory w/ $SL(2;\mathbb{Z})$ symmetry

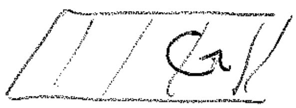
$\tau \in \mathbb{C}$, $\mathrm{Im}\,\tau > 0$

$$\begin{pmatrix}a & b\\ c & d\end{pmatrix} : \tau \longmapsto \frac{a\tau + b}{c\tau + d}$$

Cosmic strings: Greene, Shapere, Vafa, Yau

τ varies over a surface

<u>F-theory</u> : II B string theory (10 dim)

 8 - dim

$j(\tau)$ = the natural $SL(2,\mathbb{Z})$-inv. function

<u>Eisenstein series</u> $\mathbb{C}/\langle 1,\tau\rangle$

Weierstraß $\wp$-function

$$\wp(z;\tau) = \frac{1}{z^2} + \sum_{(m,n)\neq(0,0)} \left(\frac{1}{(z-m\tau-n)^2} - \frac{1}{(m\tau+n)^2} \right)$$

periods $1, \tau$

$$\wp'(z) = -\frac{2}{z^3} + \sum_{(m,n)\neq(0,0)} \frac{-2}{(z-m\tau-n)^3}$$

$\leadsto$ $\wp'(z)$

$$y^2 = x^3 - \frac{1}{3}E_4(\tau)x + \frac{2}{27}E_6(\tau)$$

$x \sim \wp$ $\quad y \sim \wp'$

$$\Delta = \frac{4}{27}\left(-E_4(\tau)^3 + E_6(\tau)^2\right)$$
$$= -2^8 \, \eta(\tau)^{24}$$
$$j(z) \sim \frac{E_4(\tau)^3}{\Delta}$$

$$y^2 = x^3 + fx + g$$

now, $f \in H^0(\mathcal{L}^{\otimes 4})$
$g \in H^0(\mathcal{L}^{\otimes 6})$

f, g now depend on parameters & base of family is 1-dim'l

<u>Kodaira</u> classified singular behavior

<u>F-theory</u>: singularities are identified

$$\begin{cases} \text{IIB w/ } n \text{ D7 branes} \leftrightarrow \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} \; I_n \\ \text{IIB w/ } m \text{ D7 branes} + \text{O7} \leftrightarrow \begin{pmatrix} -1 & m-4 \\ 0 & -1 \end{pmatrix} I^*_{m-4} \end{cases}$$

IIB w/ exotic branes $\leftrightarrow$ other Kodaira types

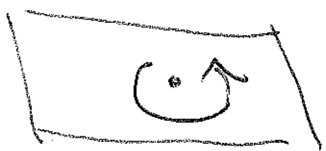
Heterotic string on T^2

24 sings

$\rightsquigarrow$ effective 8-dim theory

moduli for scalars $\mathcal{M} = SO(2,18,\mathbb{Z}) \backslash SO(2,18) / SO(2) \times SO(18)$

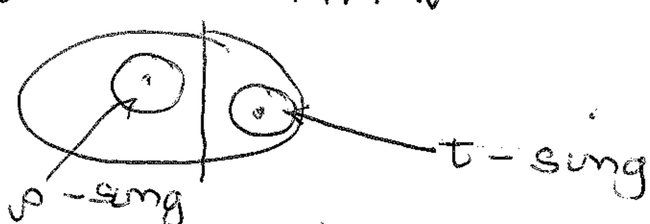
6D



The heterotic string on T^2 has a large volume T^2, then near a boundary pt of $\mathcal{M}$

$$\underbrace{Sl(2,\mathbb{Z})}_{\tau} \times \underbrace{SL(2;\mathbb{Z})}_{\rho} \times \text{Wilson line}$$

GSY $\rightsquigarrow$ HMW:



12 ρ-sing | 12 τ - sing

1) w/ McOrist x Sethi
arXiv: 1004.5992

2) w/ Malmendier
arXiv: 1102. xxxx

$$\mathcal{M} \supset \Gamma \backslash \frac{SO(2,\rho)}{SO(2)\times SO(\rho)}$$

$\underline{\rho = 2}$ (no Wilson lines) $E_8 \times E_8$ or $Spin(32)/\mathbb{Z}_2$

$\underline{\rho = 3}$ („one Wilson line") $E_8 \times E_7$ or $Spin(28) \times SU(2) / \mathbb{Z}_2$

$$\underline{y}^2 = \underline{x}^3 + fx + g$$

f, g homog of degree $8, 12$ in $\sigma \in \mathbb{P}^1$

$E_8 \times E_8$

$$\underline{y}^2 = \underline{x}^3 + A\sigma^4 x + (B\sigma^5 + C\sigma^6 + D\sigma^7)$$

$E_8 \times E_7$

$$\underline{y}^2 = \underline{x}^3 + (A_0\sigma^3 + A_1\sigma^4)\underline{x} + (B\sigma^5 + C\sigma^6 + D\sigma^7)$$

$Spin(28) \times SU(2) / \mathbb{Z}_2$

$$y^2 = x^3 + p_3(s)x^2 + \epsilon s x$$

$\downarrow$

$Spin(32)/\mathbb{Z}_2$

K3 birational to K3

$E_8 \times E_8$ $\quad$ $Spin(32)/\mathbb{Z}_2$

$E_8 \times E_7$ $\quad$ $Spin(28) \times SU(2)/\mathbb{Z}_2$

$$\rightsquigarrow \quad \begin{array}{ccc} X & \xrightarrow{Spin(32)/\mathbb{Z}_2} & \mathbb{P}^1 \\ {\scriptstyle E_8 \times E_8}\big\downarrow & & \\ \mathbb{P}^1 & & \end{array}$$

$$\begin{array}{c} Km(\mathcal{E}_\tau \times \mathcal{E}_\rho) \\ \updownarrow \ \text{2:1 cover} \\ K3 \end{array}$$

<u>no</u> Wilson line

a) $SO(2,2) / SO(2) \times SO(2) = \mathfrak{h}_\tau \times \mathfrak{h}_\rho$

$\rightsquigarrow$ description: $E_{2k}(\tau)$, $E_{2k}(\rho)$

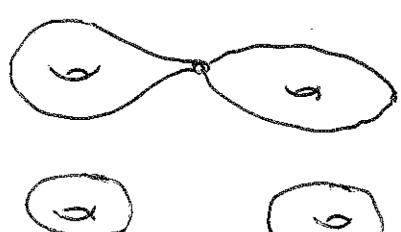
$\mathcal{E}_\tau$  $\quad$ $\mathcal{E}_\rho$ 

b) <u>one</u> Wilson line:

$$\Gamma \backslash SO(2,3) / SO(2) \times SO(3) = \mathfrak{h}_2 / \Gamma$$

↑ moduli space of genus-2 curves

→ limit

 $\mathcal{C}$

E_τ $\quad$ E_σ

$$\begin{array}{c} Kum(J(\mathcal{C})) \\ \updownarrow \ \text{2:1 cov} \\ K3 \end{array}$$

A. Kumar, Clingher–Doran

$$\psi_{2k}(\underline{\tau}) = \sum_{(C,D)\text{ distinct}} \det\left((CZ+D)\right)^{-2k}$$

invdy z

$$\chi_{10} = \frac{43867}{2^{12}\, 3^3\, 5^2\, 7 \cdot 53} \left(\psi_4 \psi_6 - \psi_{10}\right)$$

$$\chi_{12} = \frac{691}{2^{13} \cdot 3^8 \cdot 5^3 \cdot 7^2} \left(3^2 \cdot 7^2 \cdot \psi_4^3 + 2 \cdot 5^3 \cdot \psi_6^2 - 691\,\psi_{12}\right)$$

$\mathrm{Spin}(28) \times SU(2) / \mathbb{Z}_2$

$$y^2 = x^3 + \left(s^3 - \tfrac{1}{3}\psi_4 s - \tfrac{2}{27}\psi_6\right)x^2 + \left(-2^{12}\chi_{10}\, s + 2^{10}\chi_{12}\right)x$$

$$\xrightarrow[\chi_{10} \to 0]{\text{no Wilson line}}$$

$$\psi_4 \longrightarrow E_4(\tau)\, E_4(\rho)$$

$$\psi_6 \longrightarrow E_6(\tau)\, E_6(\rho)$$

$$\chi_{12} \longrightarrow \eta^{24}(\tau)\, \eta^{24}(\rho)$$

other interesting case

$$\Gamma \backslash SO(2,4) / SO(2) \times SO(4) \overset{?}{=} \Gamma \backslash SL(4) / SO(4)$$