2-Manifolds, 3-Manifolds & SUSY gauge Theories

(D. Morrison)

Starting point in physics: $A_{N-1} \leftrightarrow SU(N)$

Six-dimensional : N coincident M5-branes in M-theory in $M^{10,1}$

5+1

(M2-branes)

Indirect study: $M^{k,1} \times X^{\cancel{10}\,5-k}$ ↳ compact

$5-k=1$ $M^{4,1} \times S^1 \rightsquigarrow$ gauge field theory $G = SU(N)$

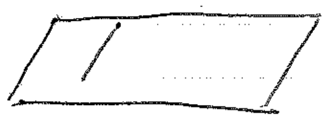
circumference ⟷ coupling of gauge theory

$M^{3,1} \times \Sigma$

Σ Riemann surface (punctured allowed) : Complicated 4d physical theory "Gaiotto theories"

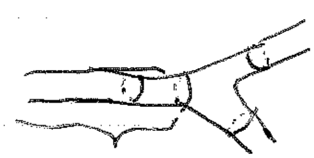
Wilson line operators

M2 ⊆ M5 M5 on $\Sigma \times \mathbb{R}^{3,1}$



M2 on $\gamma \times \mathbb{R}^{1,1}$

For appropriate metric on Σ



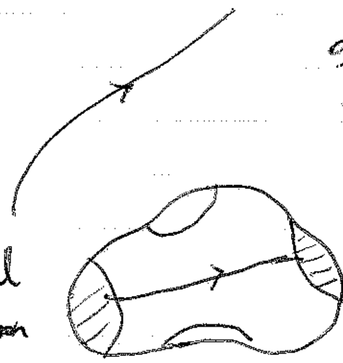
SU(2) gauge theories

pant diagram ⟷ tri-fundamental field

$3g-3$ SU(2)'s if no punctures

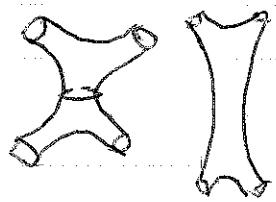
(complex)

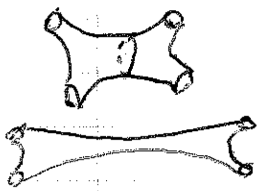
4D Theory : Wilson - 't Hooft line operators

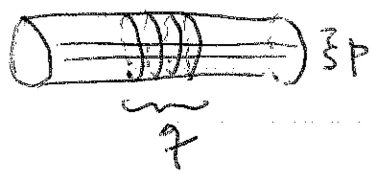


q = electric charge
p = magnetic charge

Fenchel - Neilson



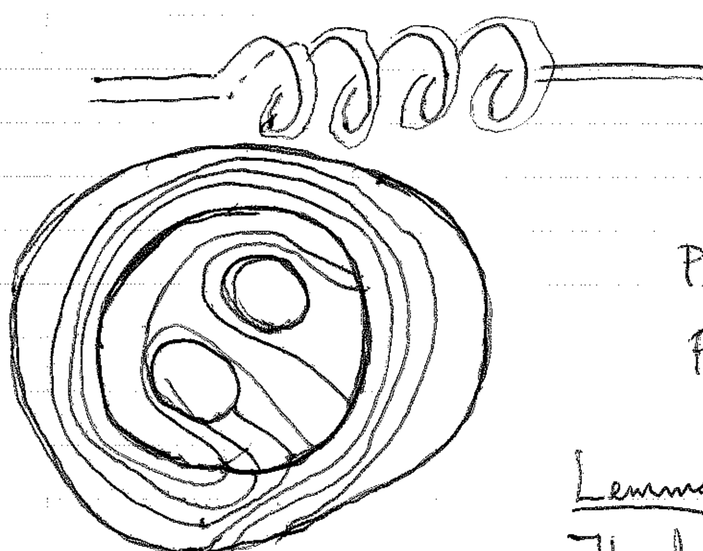




Hypothesis : γ is non-self intersecting

Data : # Crossing , # winding
near each separating geodesic

Dehn - Thurston Thm.



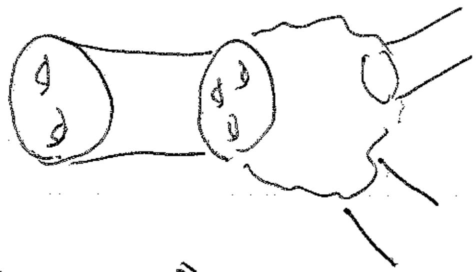
P_i even

$P_i, P_j, P_k \geq 0$

Lemma

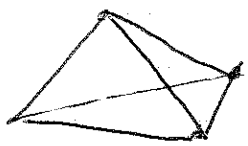
$\exists!$ homotopy class of curves
# of boundaries

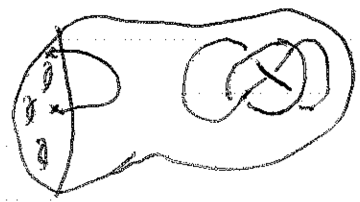
3-Manifolds



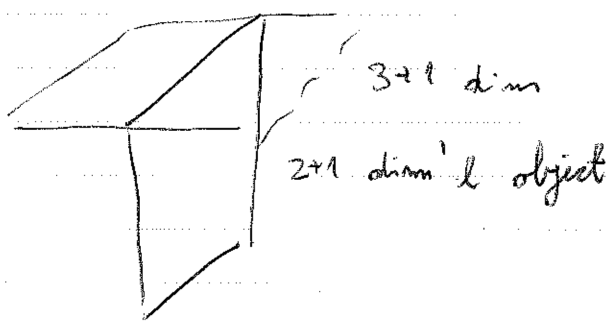
"punctures", i.e., (hyperbolic) metrics with cusp in a knot/link in M^3

decompose 3-manifold into "ideal tetrahedra"





cross-ratio



Coupling between 4D theory
3D boundary theory