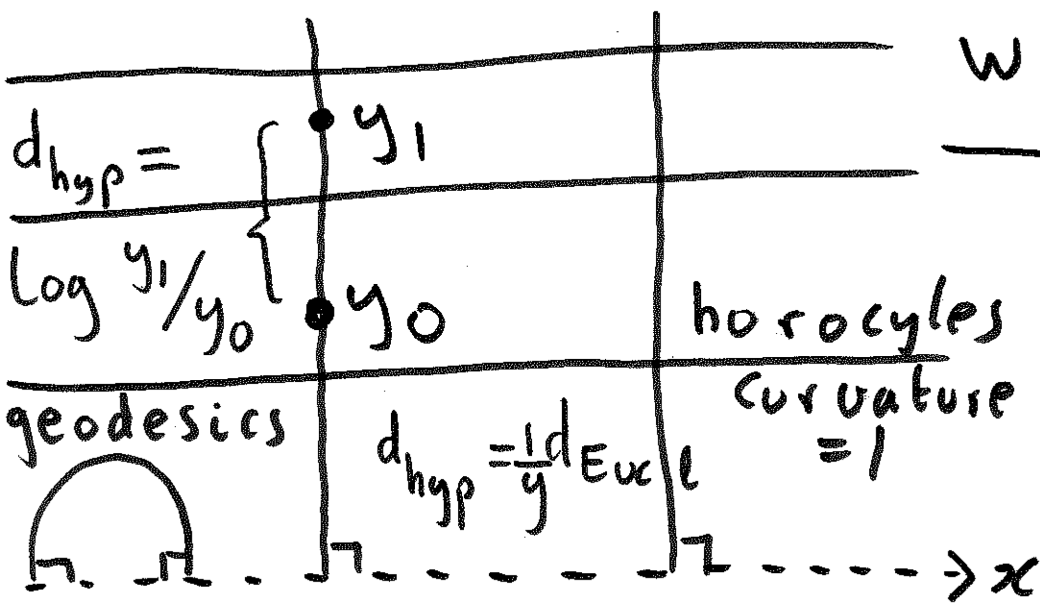
UPPER HALF PLANE

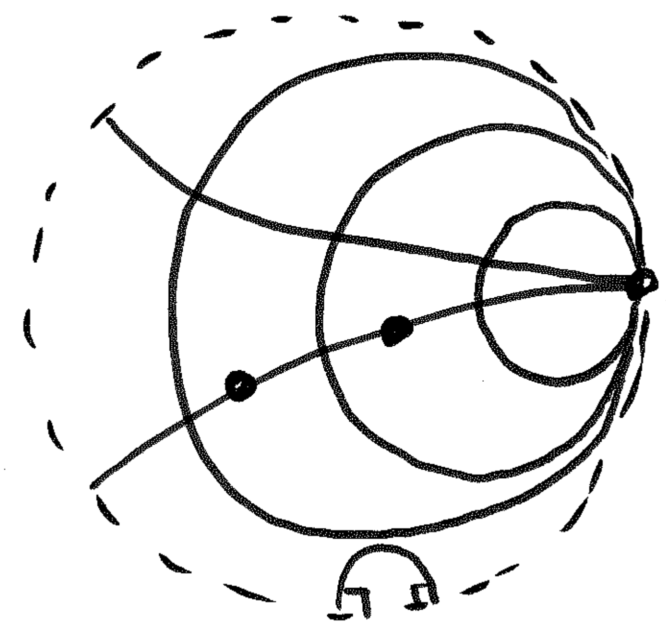
$U = \{ z : \operatorname{Im} z > 0 \}$

DISC MODEL

$D = \{ w : |w| < 1 \}$



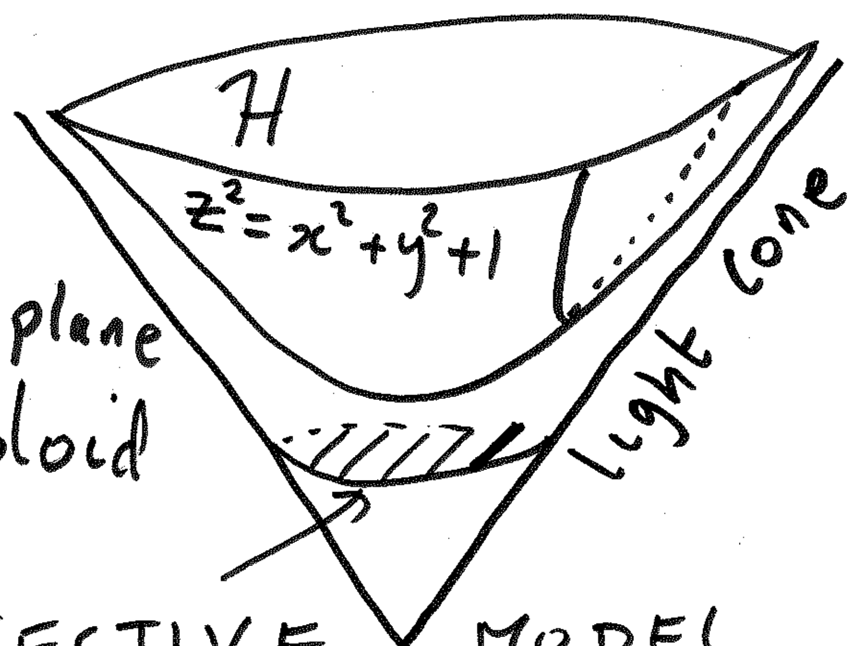
$w = \dfrac{z-i}{z+i}$ $\longrightarrow$



$\infty \mapsto 1$

Isometries $\circlearrowleft$ $\quad z \mapsto \dfrac{az+b}{cz+d}$

$PSL(2,\mathbb{R})$



geodesis $\leftrightarrow$ plane

$\mathcal{H}$ = hyperboloid model

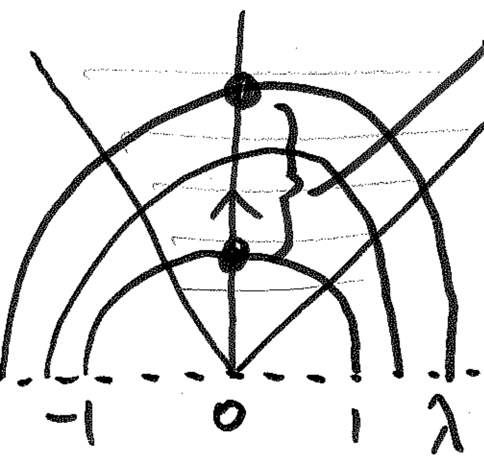
$SO(2,1)$ $\circlearrowleft$

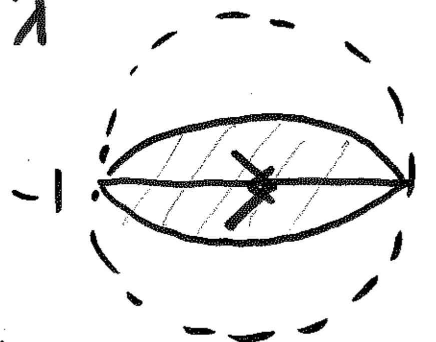
PROJECTIVE MODEL

Isometries

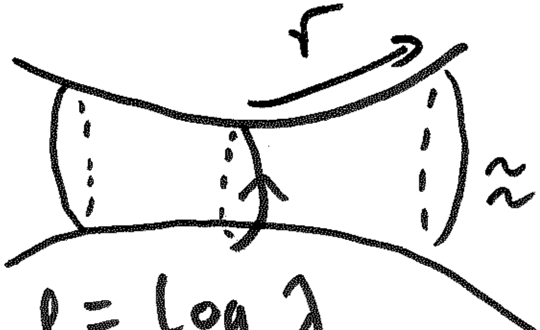
hyperbolic

$z \longmapsto \lambda z$ $\infty \longrightarrow 1$





EQUIDISTANT
NOT GEODESIC



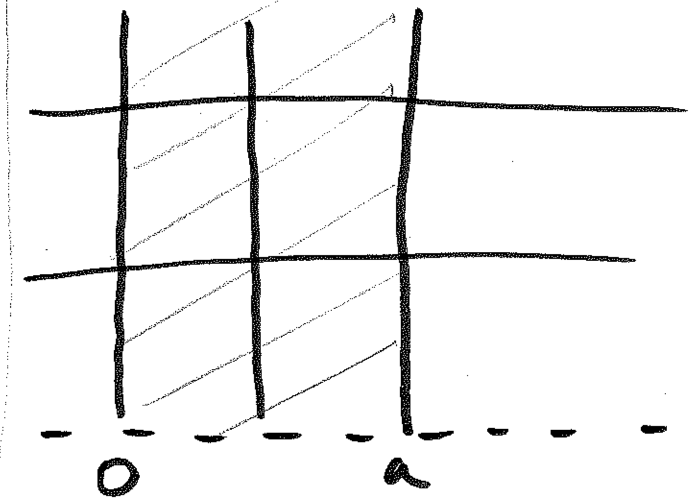
$\approx e^r \cdot \log \lambda$

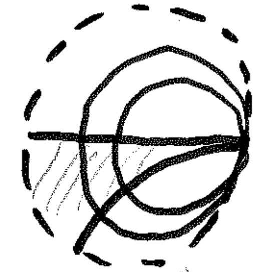
$l = \log \lambda$
$= \text{length}$

$= \mathbb{H}^2 / \Gamma$

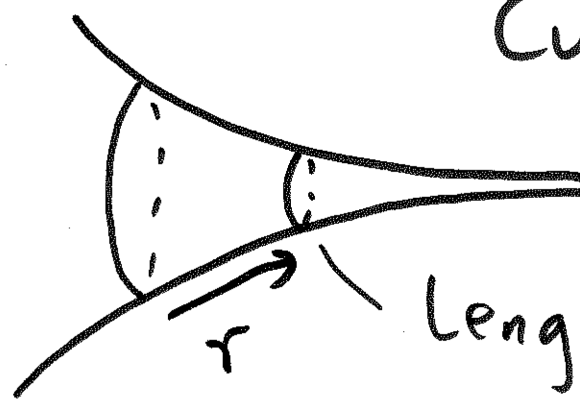
Parabolic

$z \longmapsto z + a$





Cusp

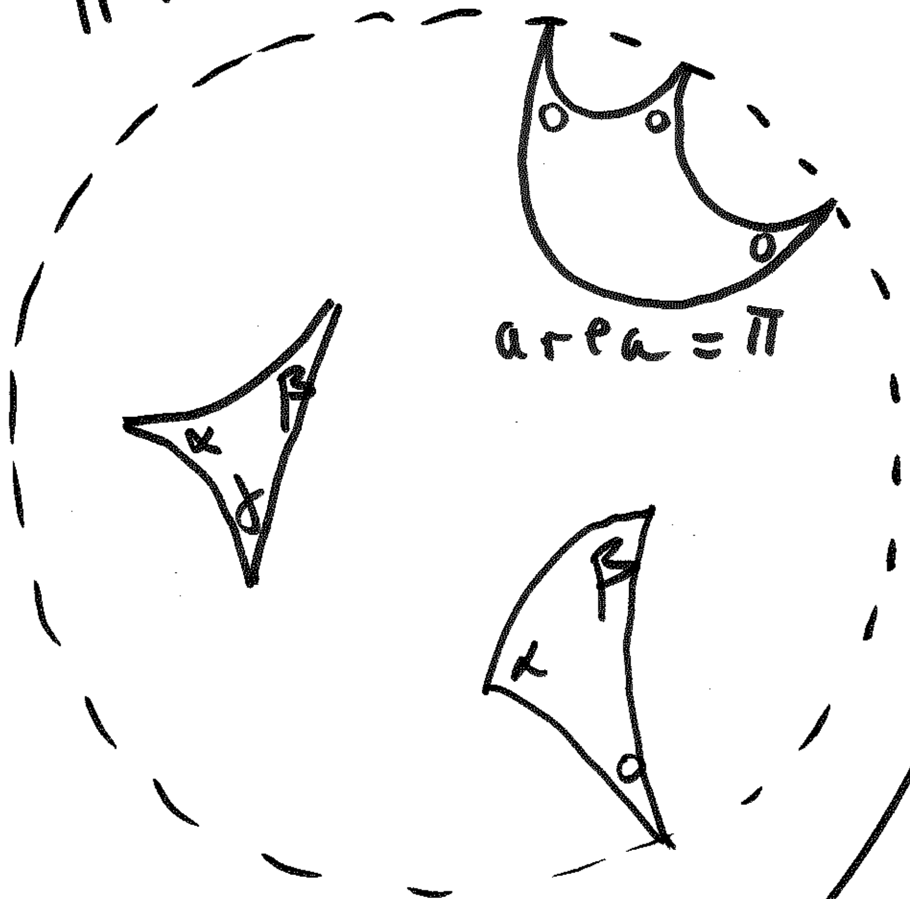


length $\approx e^{-r}$

limit $\rightsquigarrow$ infinitesimal translation along geodesic ∞ far

TRIANGLES

$\mathbb{H}^2$



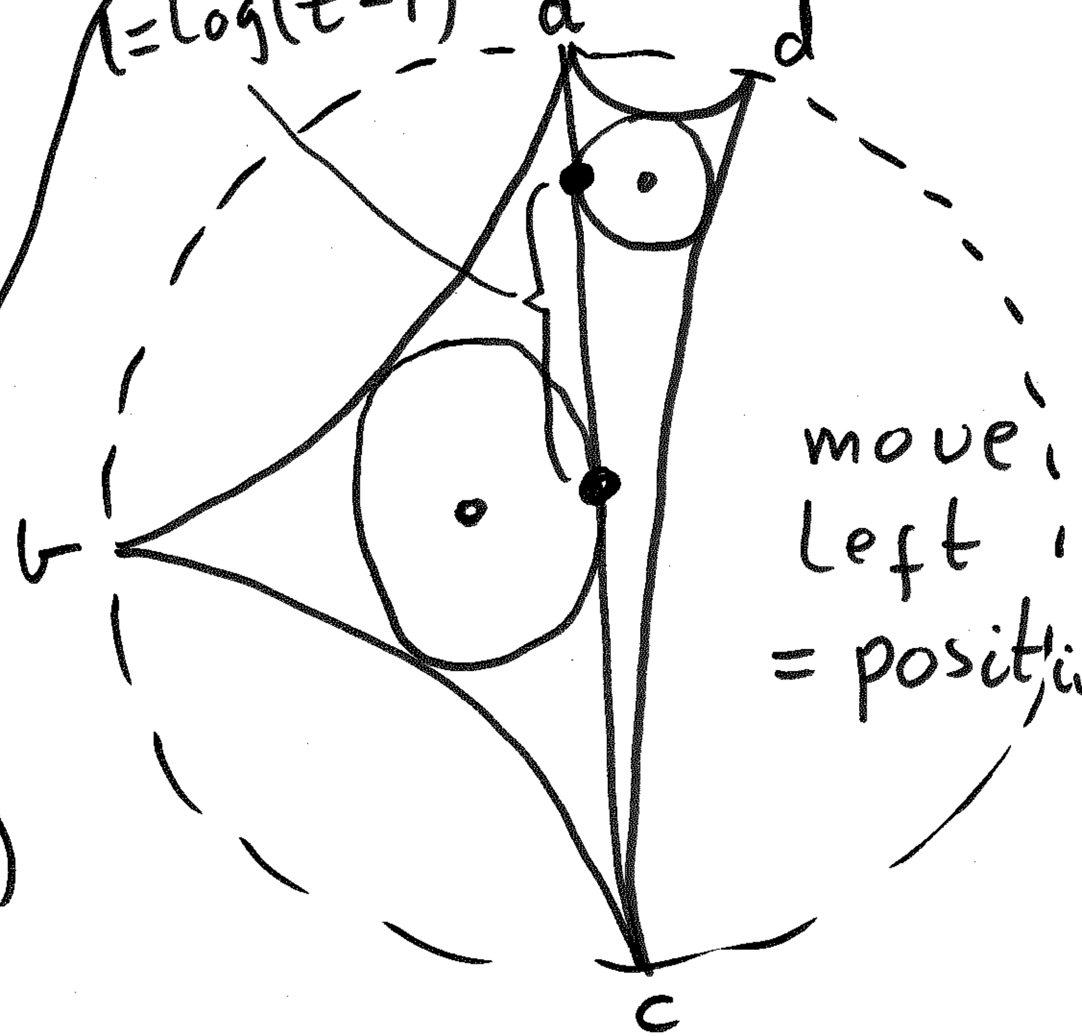
$\longleftrightarrow$ angles

$0 \leq \alpha, \beta, \gamma < \pi$

$\alpha + \beta + \gamma < \pi$

area $= \pi - (\alpha + \beta + \gamma)$

$\ell = \log(t-1)$

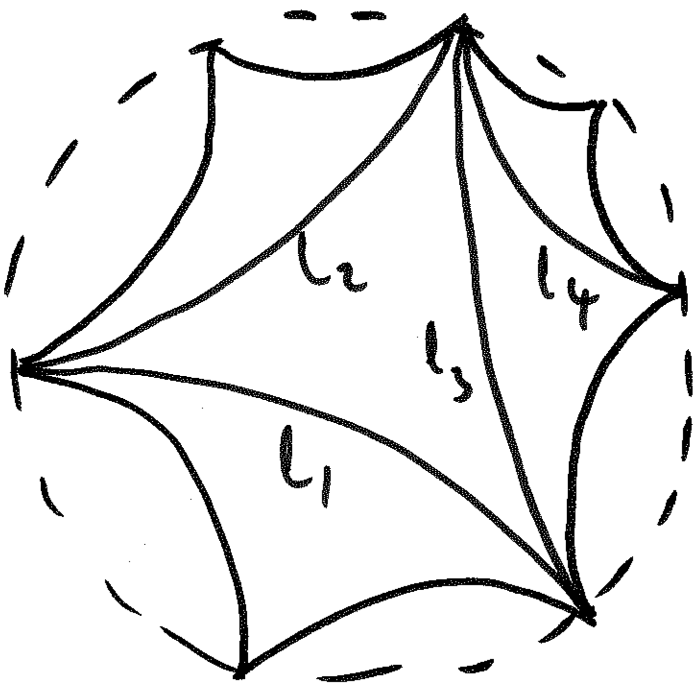


move left = positive

$t = CR(a, b, d, c) \in (1, \infty)$

Ideal n-gons

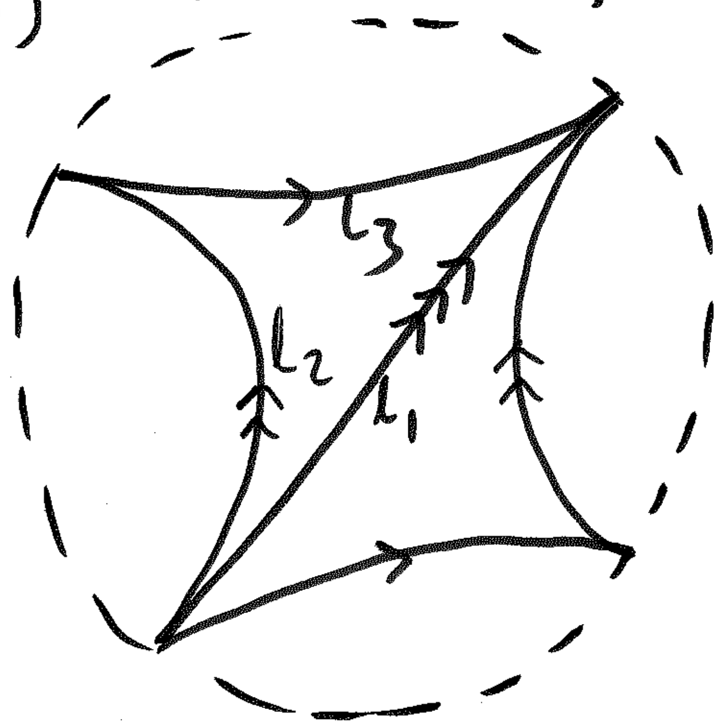
$\longleftrightarrow \mathbb{R}^{n-3}$

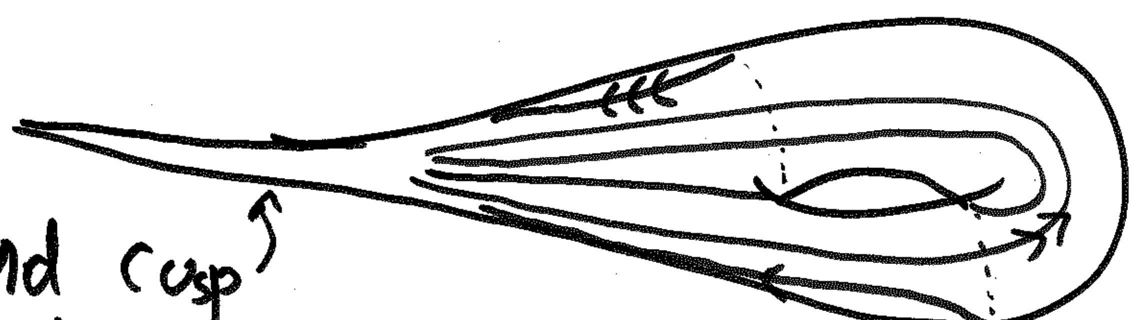


$n = 7$

T_0 = Punctured Torus

$\mathcal{T}(T_0) = \mathbb{R}^2 = \{(l_1, l_2, l_3) : l_1 + l_2 + l_3 = 0\}$





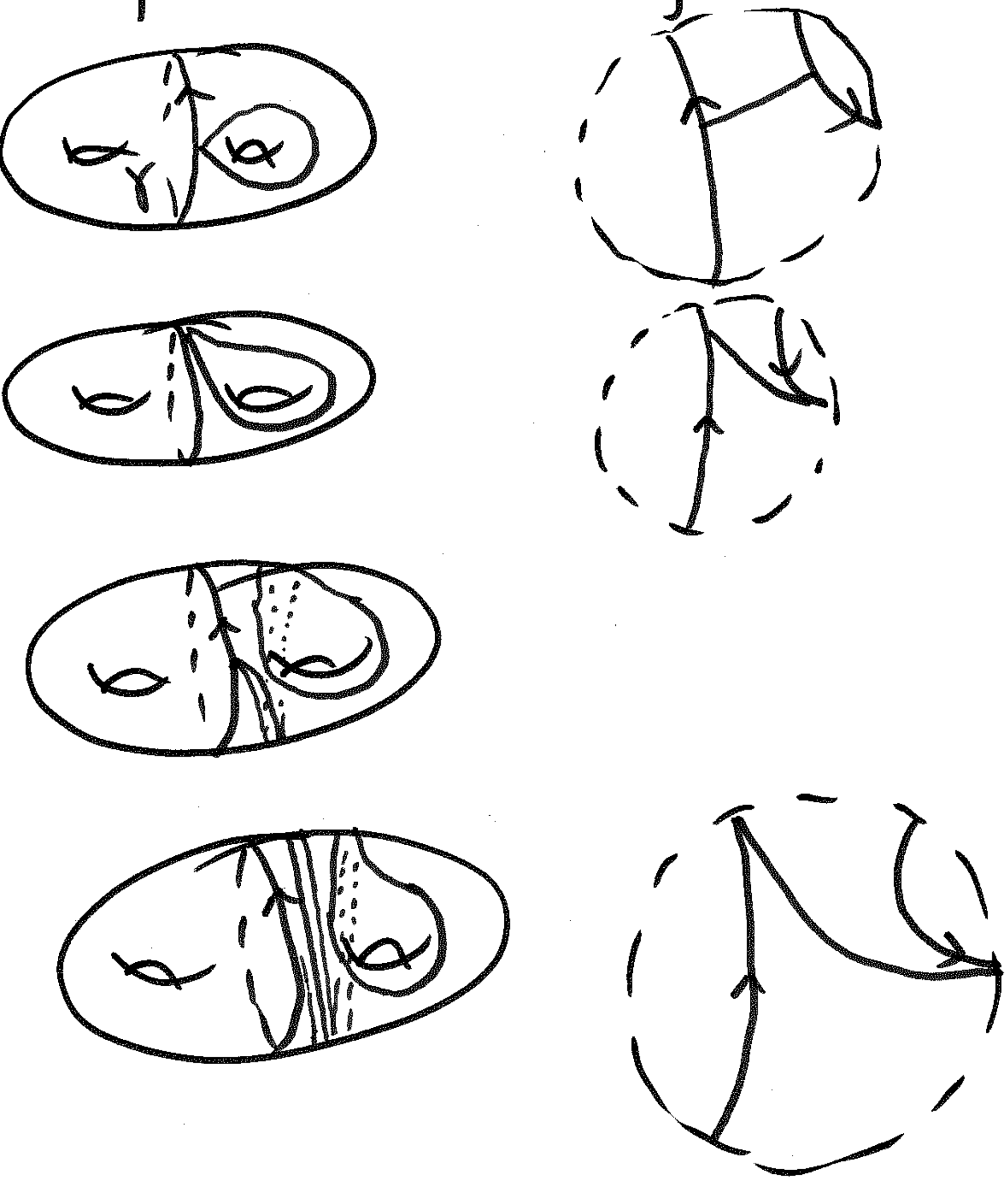
holonomy round cusp

$= 2(l_1 + l_2 + l_3)$

$0 \leftrightarrow$ parabolic

F = Closed Surface

Spin triangulation



λ = geodesic lamination

$F - \lambda = \bigcup$ ideal triangles

$\to \lambda = \gamma \cup$ arcs

$F = \gamma \cup$ (2 punctured tori)

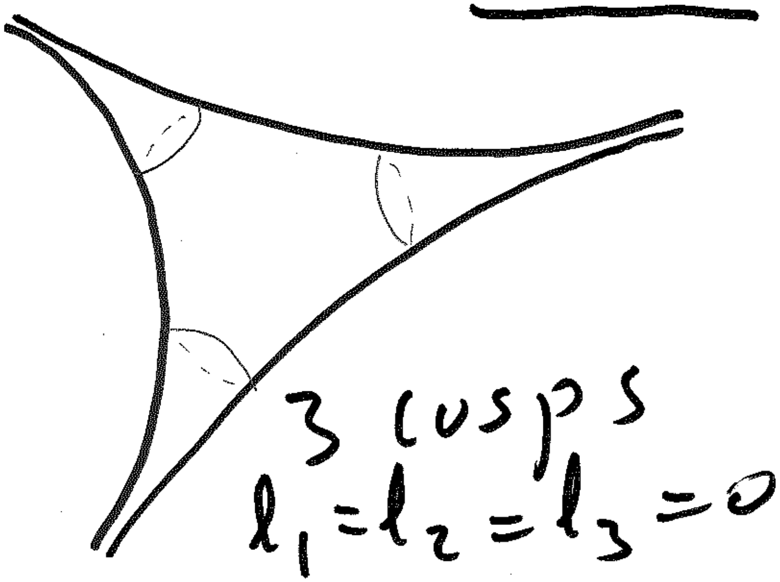
$l_1, \dots, l_6$

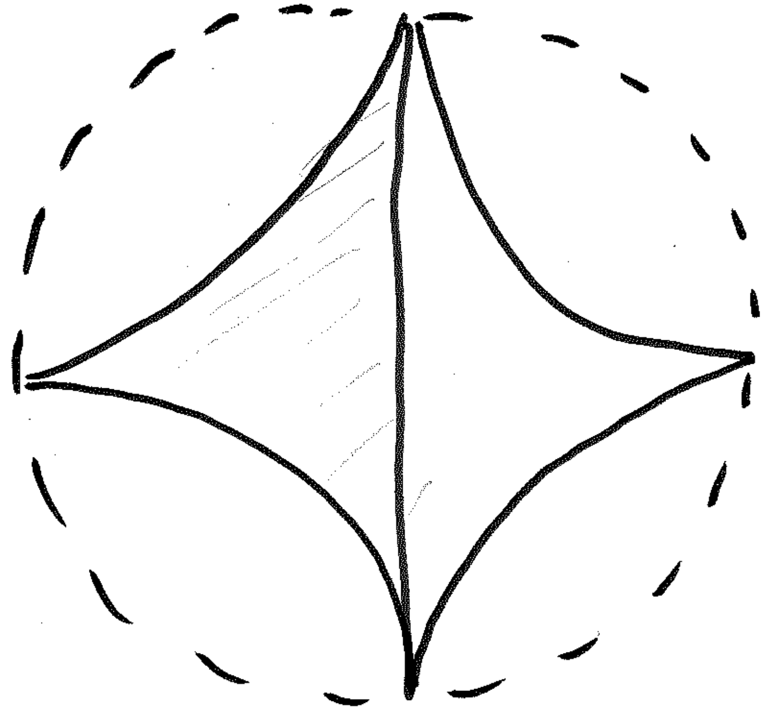
$l_1 + l_2 + l_3 = l_4 + l_5 + l_6$

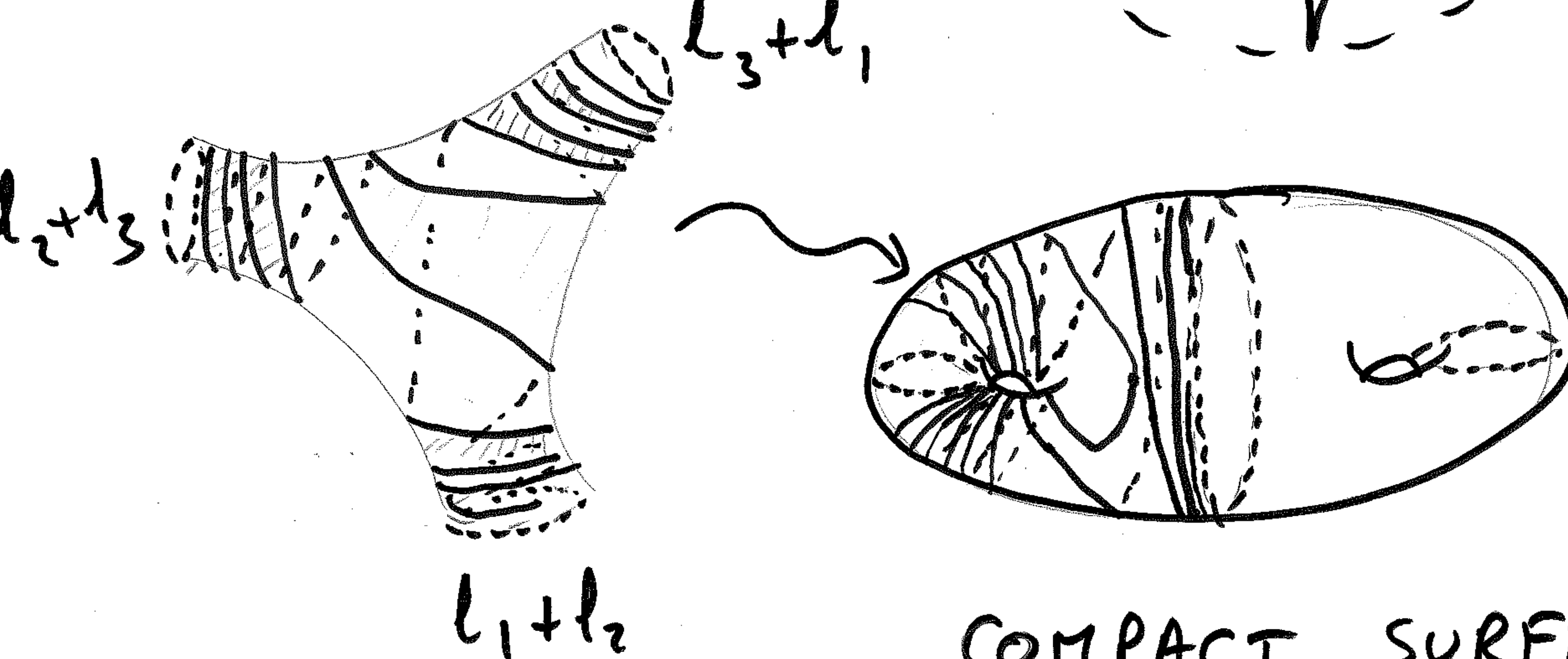
+ twist parameter

$\mathbb{R}^6$ $\quad$ $\mathbb{R}^{3 \cdot |\chi(F)|}$

PANTS

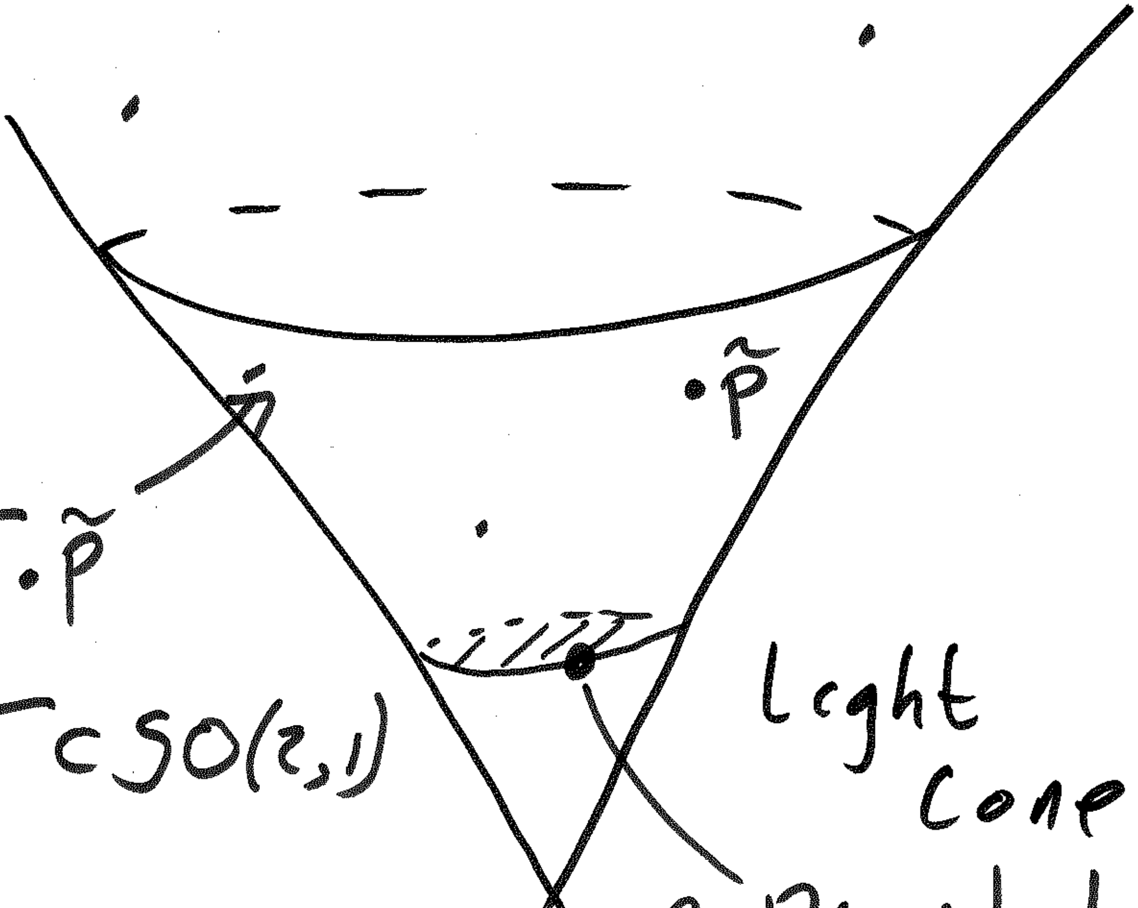






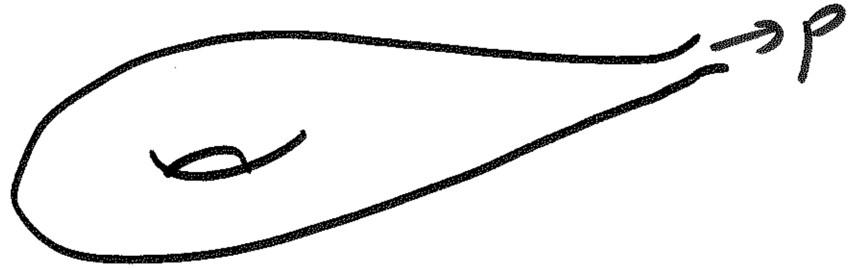
COMPACT SURFACE

Epstein - Penner construction



$\Gamma \subset SO(2,1)$

$\mathbb{H}^2/\Gamma$ =



hyperbolic
one cusp

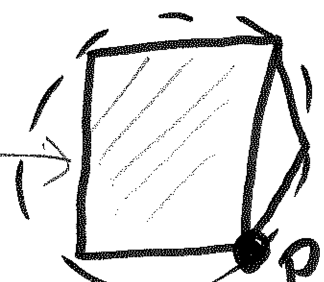
$\boxed{\text{Canonical cell decomp}}$

complete invt.

Convex hull$(\Gamma.\tilde p)$ = ∞ sided polytope

project edges

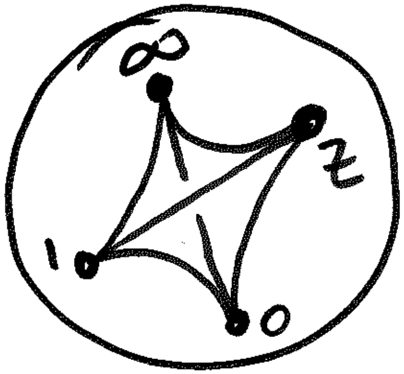
Convex ideal Polygon



Projective model

Ideal triangulations of 3 Manifolds

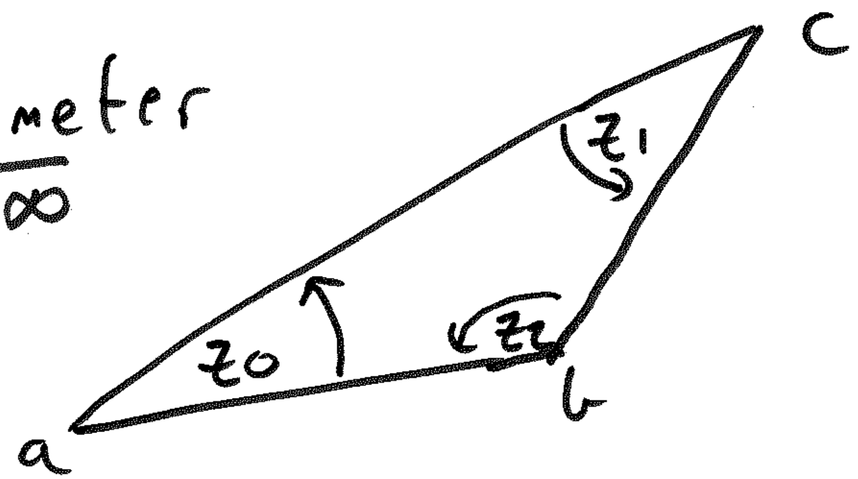
$\mathbb{H}^3$



$\hat{\mathbb{C}} = \partial_\infty \mathbb{H}^3$

$z \in \hat{\mathbb{C}} - \{0,1,\infty\} \longleftrightarrow$ ideal 3-simplex

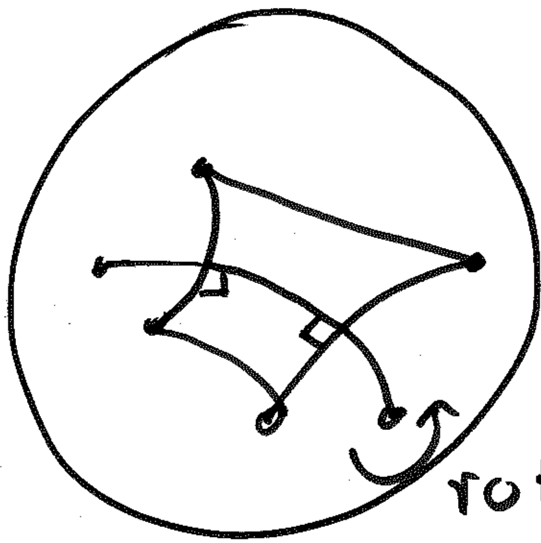
z ← edge parameter on $\overline{0\infty}$



$$z_0 = \frac{c-a}{b-a} \quad \text{etc}$$

$$z_0 = z \qquad z_1 = 1 - z^{-1} \qquad z_2 = \frac{1}{1-z}$$

- Same parameter on opposite edges

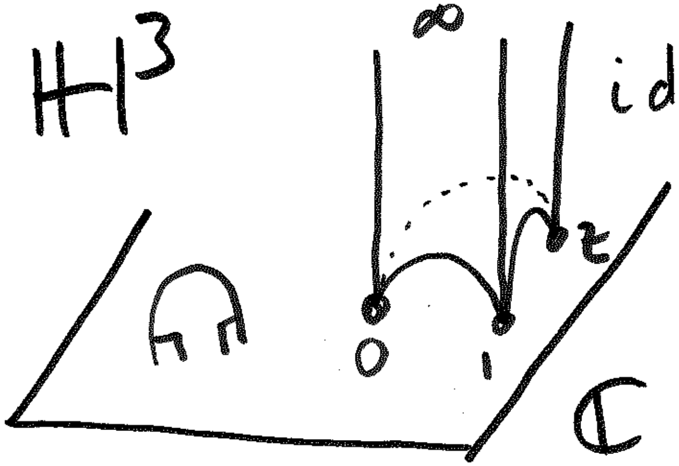


rotate π

Edge Equation

$$\prod_{\substack{\text{round}\\\text{edge}}} z = 1$$

$\mathbb{H}^3$



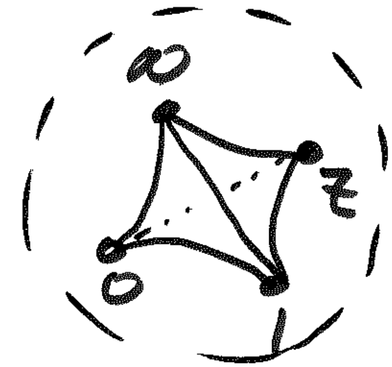
$\circlearrowleft$ $PSL(2,\mathbb{C})$

ideal 3-simplex

$\updownarrow$

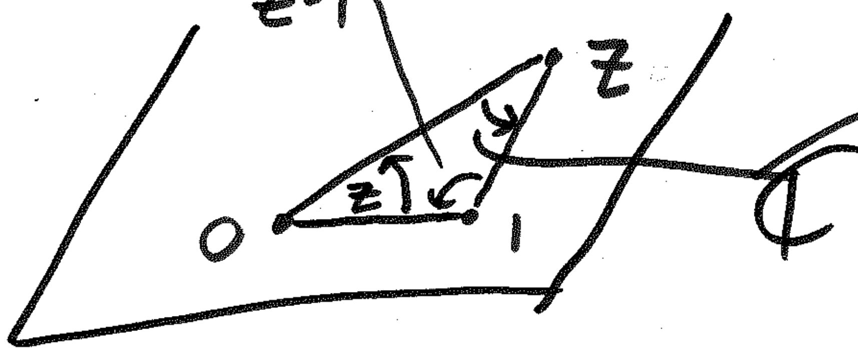
$z \in \mathbb{C} - \{0, 1, \infty\}$

$\hat{\mathbb{C}}$

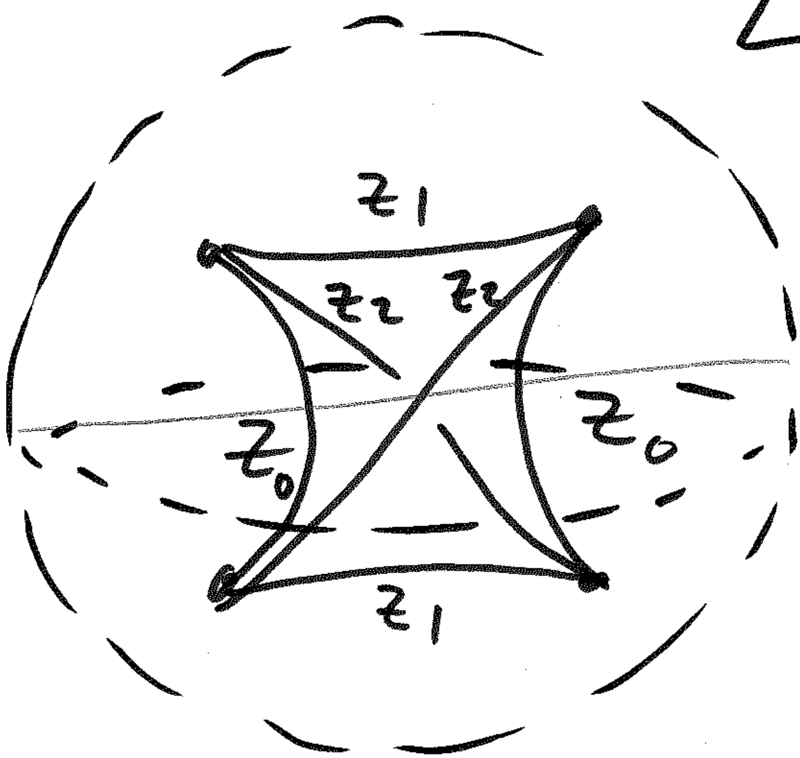


D^3

$\frac{0-1}{z-1} = \frac{1}{1-z} = z_2$



$z_1 = \frac{1-z}{0-z}$
$= 1 - z^{-1}$



$\frac{1}{2}$ rotate

$\hat{\mathbb{C}}$

$z_0 z_1 z_2 = -1$

edge parameter

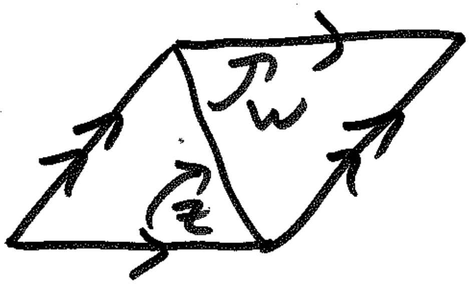
Cusp Completeness Condition

Solution gluing equations $\longrightarrow$ Euclidean Similarity Structure

$z \mapsto a z + b$

M^3 <u>complete</u> $\Longleftrightarrow$ Euclidean structure

$\Longleftrightarrow$ $a = 1$



$zw = 1$ general $\prod z_i = +1$

$\mu =$ translation $z \mapsto z + m$

$\lambda \mu = \mu \lambda \Rightarrow \lambda =$ translation

T^2