Gauge Theories Labeled by three-manifolds (D. Morrison)

1108.9589

Idea: $(M^3, \partial M^3) = \bigcup_i (T_i, \partial T_i)$

- hyperbolic

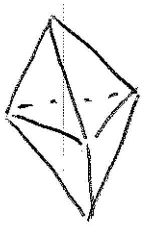
$(T_i, \partial T_i) \longleftrightarrow$ field theory

gluing procedure

Additional data on boundary:

$\{$ flat $SL(2,\mathbb{C})$ connection on $\partial M^3 \} /$ gauge equivalence

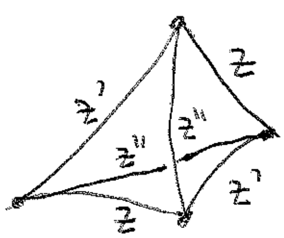
$\cap|$



complexification of space:

$\mathcal{P}_{\partial M}$ = phase space on ∂M

$\mathcal{P}_{\partial T}$ = phase space on ∂T



$z \in \mathbb{C}P^1$

$\boxed{z \cdot z' \cdot z'' = 1}$

$\cancel{z'' + z' + 1 = 0}$ (?)

$z, z', z'' \in \mathbb{C} \setminus \{0, 1\}$

$\mathcal{P}_{\partial T} = \{ (z, z', z'') \in (\mathbb{C}^* \setminus \{1\})^3 \mid z z' z'' = -1 \} \cong (\mathbb{C}^* \setminus \{1\})^2$

symplectic structure

$\omega = \frac{1}{\hbar} dz \wedge dz'$

trivial holonomy

$\mathcal{L}_{\partial T} \subseteq \mathcal{P}_{\partial T} \longrightarrow$ flat $SL(2,\mathbb{C})$ on ∂T, unipotent holonomies on punctures

$\mathcal{L}_{\partial T} = \{ z + (z')^{-1} = 1 \}$ = Lagrangian submanifld of the symplectic space

<u>Field theory</u>

Background fields, dynamical fields
↑
couplings, exp. values
fill out
$\mathcal{P}_{\partial T}$

Need a polarization in the symplectic space $\mathcal{P}_{\partial T}$
i.e. a choice of conjugate coords. in space

polarizations for ∂(Tetrahedron)

coord.	mom.
z	z'
z'	z''
z''	z

$SL(2,\mathbb{Z}) = Sp(2,\mathbb{Z})$-transformations give other choices.

$$z'' = \frac{1}{z z'} \qquad \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} \log z \\ \log z' \end{pmatrix}$$

Witten ~ 2002

$SL(2,\mathbb{Z})$ action on a class of 3d field theories, coupled to a background field

U(1) theory, bckgund. A

$$T: \mathcal{L} \to \mathcal{L} + \frac{1}{4\pi} A \wedge dA$$

S: make A dynamical, add new back. field

$$\mathcal{L} \to \mathcal{L} + \frac{1}{2\pi} A_{new} \wedge dA$$

E-M duality on 4d theory: what effect on 3D boundary conditions?

S^2 = charge conj. on background field

$(ST)^3 = id$

Idea of paper:

1) Write a SUSY version of $SL(2,\mathbb{Z})$ action.
2) Associate $\mathcal{N}=2$ SUSY 3d CFT to polarized $(T, \mathcal{P}_{\partial T})$
(coupled to a backgnd)

→ single hypermultiplet charged under $U(1)$ + CS term of level $\frac{1}{2}$

glue to $U(1)^N$ theory, add new superpotential $W = \sum_I \mathcal{O}_I$
I = internal edges in 3-manifold.