# F-theory Lecture 2. Dave Morrison

* elliptic condition $\Rightarrow \exists$ section

$$\sigma \circlearrowright \begin{matrix} X \\ \downarrow \pi \\ S \end{matrix}$$

$S$ = base space, alg. variety

* general fiber a curve of genus one.

<u>Weierstrass Model</u>

$$\begin{matrix} \mathcal{L} \\ \downarrow \\ S \end{matrix} \text{ line bundle}$$

$f \in H^0(S, \mathcal{L}^{\otimes 4})$ $g \in H^0(S, \mathcal{L}^{\otimes 6})$

$\{y^2 = x^3 + fx z^4 + g z^6\}$ $x \in H^0(S, \mathcal{L}^{\otimes 2})$ $y \in H^0(S, \mathcal{L}^{\otimes 3})$

$\cap$

$\mathbb{P}(\mathcal{O} \oplus \mathcal{L}^{\otimes 2} \oplus \mathcal{L}^{\otimes 3})$

$S \supset U$ = open nghbd of a generic point on the singular locus of $\pi$

* localize the sing, yield DVR $z$

$\{z=0\} \longleftrightarrow$ component of singular locus of $\pi$

<u>Tate's alg. in char 0</u>

* goal: to understand codim 1 sing on $S$.

$$\frac{\partial}{\partial y}(-y^2 + x^3 + fx + g) = -2y = 0$$

$$\frac{\partial}{\partial x} \Rightarrow \Delta = 4f^3 + 27g^2 \text{ locates the singularity}$$

(away from 0's of $f$)

at $\Delta = 0$, $f = 0 \Longleftrightarrow g = 0$.

components ① $f, g$ not generically 0 on $\Sigma$ — comp. of $\Delta$ ②

② $f|_\Sigma = 0$ $g|_\Sigma = 0$

change coordinates

$y^2 = x^3 + ux^2 + vx + w$ st. sing point at $(0,0)$.

$[u = -\frac{9g}{2f}]$ $z|v$ and $z|w$.

$u, v, w$ no longer globally defined

$\Delta = 4u^3w - u^2v^2 - 18uvw + 27w^2$

$v = zv_1$ $w = zw_1$ $f, g \neq 0 \Rightarrow z \nmid u$. (z does not divide u)

$m = \underset{z=0}{\mathrm{ord}}(\Delta)$ $\Delta = 4u^3w_1z + O(z^2)$

$\Rightarrow m \geq 1$. if $m \geq 2 \Rightarrow z|w_1$.

$m=1$: total space is non-singular

$m \geq 2$: $z|w_1$, write $w$ as $z^2w_2$. singular

$ux^2 + v_1zx + w_2z^2$

* simple blowup of origin $\Rightarrow$ non/s deg 2 in $\mathbb{P}^2$

if $m \geq 3$ $\tilde{y}^2 = \tilde{x}^3 + \tilde{u}\tilde{x}^2 + \tilde{v}_2z^2\tilde{x} + \tilde{w}_3z^3$

$\tilde{y}^2 - \tilde{u}\tilde{x}^2 =$ cubic. if $\sqrt{\tilde{u}}$ exists in my ring, then $(\tilde{y} + \sqrt{\tilde{u}}\,\tilde{x})(\tilde{y} - \sqrt{\tilde{u}}\,\tilde{x})$

if not: the individual lines are not defined over the field.

$I_2$ case

onto $I_3$ case

$\sqrt{u}$ exists $\Rightarrow$ splits $SU(N)$

$\sqrt{u}$ D.N.E. $\Rightarrow$ non-split $Sp(N)$

$z^{\frac{m}{2}} | v$ $\quad z^n | w$ $\quad \Delta = \boxed{\ } z^m + O(z^{m+1})$

1 or two terms

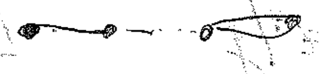
$\underline{I_m}$

chain of $m-1$ curves

$\underline{\text{Lie algebra}}$ dual graph of resolution of sing = Dynkin diagram

$m-1$

* $\underline{\text{aut.}}$ exchange ends



$B_k$

$\underline{\text{Case II}}$ back to Weierstrass assume $z | f$, $z | g$

$\Delta = 4f^3 + 27g^2 \Rightarrow \Delta = 27 g_1^2 z^2 + O(z^3)$

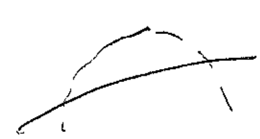
$\text{II}$: $z^2 \nmid g$

$\text{III}$: $z^2 \nmid f$ $\quad z^2 | g$

$\text{IV}$: $z^2 | f$, $z^2 | g \Rightarrow \Delta = 27 g_2^2 z^4 + O(z^5)$

"$z^2 | f$ $z^3 | g$ interesting case

~~*non singular tot~~



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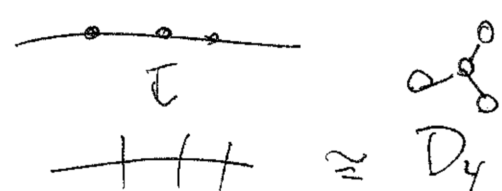
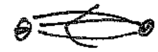
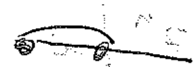
$\underline{\text{3 distinct roots}}$



$I$ $\quad \cong D_4$

no roots over field 



3 roots $D_4$ 

$$\sqrt{\left(\frac{\Delta}{z^{2m+1}}\right)\left(\frac{-2zf}{9g}\right)^{3}}\Bigg|_{z=0}$$

$$\sqrt{\frac{-\Delta}{z^{2n+2}}\left(\frac{-2zf}{9g}\right)^{2}}\Bigg|_{z=0}$$

$z^3 | f$ $z^4 | g$ $\longrightarrow$ $\Delta = 4f^3 + 27g^2$ $IV^*$

$z^5 | g$ $III^*$ $z^4 | f$ $II^*$

$\boxed{z^4 | f \quad z^6 | g}$ $\longrightarrow$ "STOP" modify $f \mapsto \frac{f}{z^4}$

$g \mapsto \frac{g}{z^6}$

$x, y \mapsto$ approp. powers

$\Rightarrow$ new W.S. w/ new $\mathcal{L} = \mathcal{L} \otimes \mathcal{O}(-\Sigma)$

"singularities are so bad, you should have used another bundle"