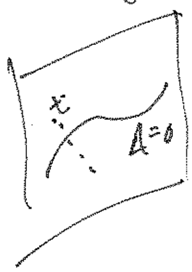
F-theory Dave Morrison Lecture 3. matter

References: Katz-Vafa "Matter & Geometry"
Katz-Morrison "Gorenstein + ..."

Kodaira - Tate: $y^2 = x^3 + fx + g$ $f \in H^0(\mathcal{L}^{\otimes 4})$ $g \in H^0(\mathcal{L}^{\otimes 6})$

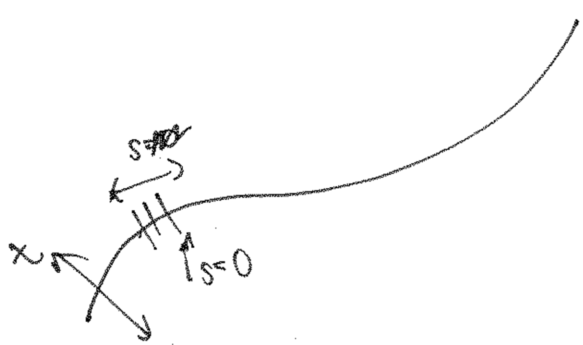
$\Delta = 4f^3 + 27g^2$

slice
$t = 0$ singular
$t \neq 0$ non-singular



* orders of vanishing of f, g, Δ ⇒ specify singularity

at $s = t = 0$, singularity is further enhanced.



"Simultaneous resolution of $\underbrace{\text{rational double points}}_{\text{ADE singularities}}$"

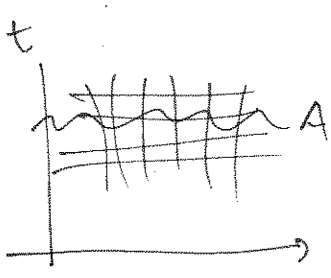
sim. res: Grothendieck, Brieskorn

Assumption along $s = 0$, t arbitrary, we have a minimal Weierstrass model i.e. either

$\mathrm{ord}_{t=0} f|_{s=0} \leq 3$ or $\mathrm{ord}_{t=0} g|_{s=0} \leq 5$

non-minimal points correspond to extremal transitions and can be analyzed by blowing up the base.

local model



at $s = 0$ have RDP.
what deformations does it have.

* local singularities on surfaces. ask questions up to holomorphic change of coords.

$f(x,y,t)=0 \longmapsto f(x,y,t)+\sum a_I \vec{x}^{I}$ $\qquad f+\left(\frac{\partial f}{\partial x,y,t}\right) g$ is trivial

Examples $\quad A_n$: $\cancel{x^2+y^2}\; xy + t^{n+1} + a_2 t^{n-1} + a_3 t^{n-2} + \dots + a_{n+1}$

↳ deformations divisible by x, y or t^n are trivial, $\Rightarrow$ finite

D_n: $x^2+y^2t+t^{n-1}+\dots$

Bl A_n review

$xy+t^{n+1}$

$\vdots$

$\tilde{x}\tilde{y}+t^n$

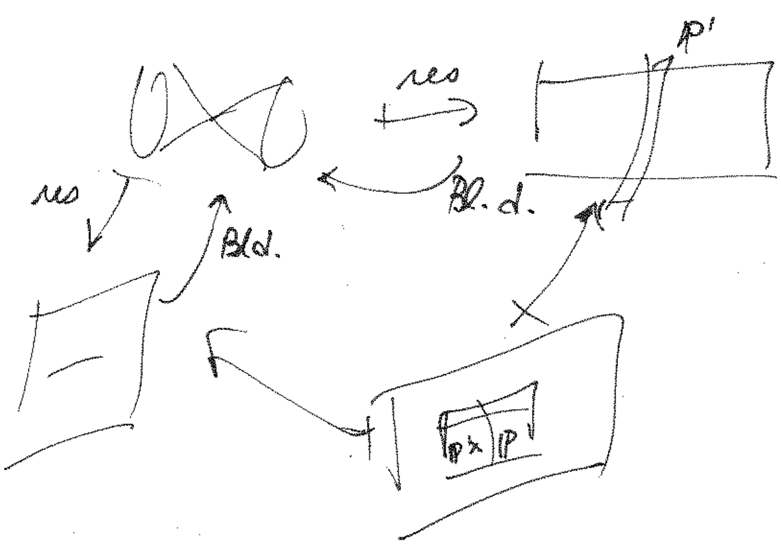
E_6: $x^2+y^3+t^4+\dots$

E_7: $x^2+y^3+yt^3+\dots$

E_8: $x^2+y^3+t^5+\dots$

$xy+t^2-s^2 = xy+(t+s)(t-s) \Rightarrow \exists$ a small resolution

blowup



* family of surfaces parametrized by s which are smooth at every point.

* have singularity + def. param.

Thm: Brieskorn + Grothendieck

i) def. space of ADE is a nbd. of the origin in $\mathbb{C}^n / W(R_n) \cong \mathbb{C}^n$

$X \mapsto \mathbb{C}^n / W$ = "versal" family

form

$$\begin{array}{ccc} \tilde{X} & \longrightarrow & X \\ \downarrow & & \downarrow \\ \mathbb{C}^n & \longrightarrow & \mathbb{C}^n/W \end{array}$$

A_1): $xy + t^2 + a_2$ $\quad a_2 \in \mathbb{C}/W$ $\qquad W = \mathbb{Z}/2 \quad \mathbb{C} \mapsto \mathbb{C}/W$

$s \mapsto -s^2$

A_n) $xy + t^{n+1} + a_2 t^{n-1} + \dots + a_{n+1}$ $\quad a_2, \dots, a_{n+1}$ = alt sym

$s_1, \dots, s_{n+1}$ st $\sum s_i$ $\qquad W = S_{n+1}$ in $\mathbb{C}^n$

$\hookrightarrow xy + \prod_{i=1}^{n+1} (t + s_i)$

$x = t + s_1 = 0 \;\vdots$
$x = t + s_2 = 0 \;\vdots$

$W(A_n) = S_{n+1}$

$\hookrightarrow$ Weyl group

$\mathbb{C}^n / W(R_n)$ coords are W-inv functions, the Casimirs

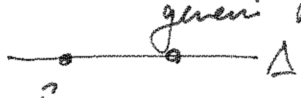
<u>E_6)</u> $x^2 + y^3 + t^4 + a_2 y t^2 + a_5 y t + a_6 t^2 + a_8 y + a_9 t + a_{12}$

$\Rightarrow \deg x = 6 \quad \deg y = 4 \quad \deg t = 3$

$\hookrightarrow$ def'ns $\quad \underset{0}{1}, \underset{3}{t}, \underset{6}{t^2} \quad \underset{4}{y}, \underset{7}{yt}, \underset{10}{yt^2}$

$W(D_n) = \mathbb{Z}_2^{n-1} \rtimes S_n$

generic R_n



special R'_{n+1} different root system.

$E_6 \to R'_6$ some def give D_5, some A_5

↳ how do we tell which one?

$R'_{n+1} = A_{n+1}$ $xy + t^{n+2} \mapsto \boxed{xy + t^{n+2} + s t^{n+1}}$

$R_n = A_n$

BR $\tilde{x}, \tilde{y} \mapsto x, y$

$\boxed{xy + t^n + s t^{n-1}}$

$\boxed{xy + t + ts}$ $\boxed{xy + t^2 + st}$

* Witten: M-th F-th

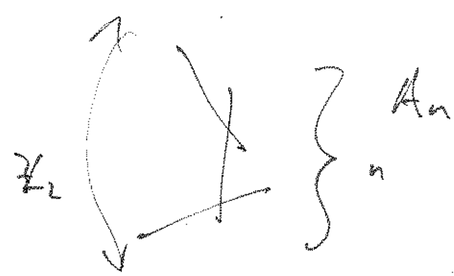
half-hypers

* half hyper hyper is $\mathbb{H}$-rep sometimes $\mathbb{H} = \mathbb{C}, \bar{\mathbb{C}}$

ex

* $R'_{n+1} = D_{n+1}$ $R_n = A_n$

D_{n+1} anti sym. rep.



A_n n curves

antisym (but of Sp)

either: $\mathbb{Z}_2$ is trivial, conifolds, sim. resolve

$\mathbb{Z}_2$ non-triv, sim. res. not possible

* Cladov on symmetries