

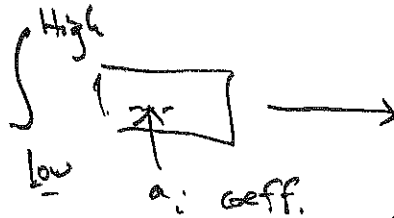
Algebraic Geometry and 6D Superconformal

Field theories

- Conformal field theories:

gauge field theories:

scale dependent (Wilsonian renormalization)



scale invariant theories



\rightarrow all theories

semigroup "Renormalization Group"

High energy "ultraviolet" UV:

Free field theory in UV } Lagrangian
+ small perturbation } description.

Fails in spacetime dim > 4 . (free theory fails)

Start with something from a gravity theory
in higher dimensions:

String theories & M-theory

10

$$C^4 \times R^{1,5}$$

11

$$C^5 \times R^{1,5}$$



Heterotic $E_8 \times E_8$ string theory

10D theory gravity + Yang-Mills
 τ
 g_{ij}
 $E_8 \times E_8 \times \mathbb{Z}_2$
 $A_i, e_8 \oplus e_8.$

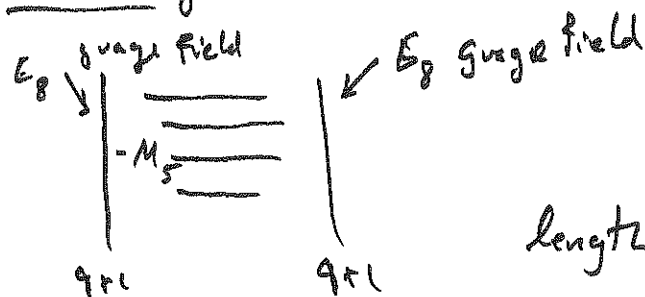
10D theory on $K3$ surface, X
 (Ricci flat Kähler metric)

bundle $E_8 \times E_8$ structure group
 holo. sections

Limiting singular bundle:
 pointlike singularity at $P \in X$



M-theory 11D



length \rightarrow het $E_8 \times E_8$ theory
 \sim string coupling

M9-brane 9+1

M5-brane 5+1

M2-brane 2+1

M5 brane dissolved in M9-brane

Put curvature sing. of bundle

at a mild sing. of \mathbb{R}^3 space.

$$\mathbb{C}^2/\Gamma, \quad \Gamma \subseteq \text{su}(2) \rightarrow \text{so}(3)$$

ADE, A_n, D_n, E_6, E_7, E_8 .

Dual description in terms of "F-theory"

with

del Zotto, Heidebrunn,
Park, Rudelius, Vafa

1212.5746

1412.6526

1502.05405

1505.00009

type IIB string theory:

• complex scalar field τ , $\text{Im}(\tau) > 0$

• $\text{SL}(2, \mathbb{Z})$ -symmetry

$$\tau \mapsto \frac{a\tau + b}{c\tau + d}$$

• D7-branes



$$\tau \mapsto \tau + 1$$

$$y^2 = x^3 + fx + gy$$

$$f = f(s, t)$$

$$g = g(s, t)$$

determine τ up to

$$\text{SL}(2, \mathbb{Z}).$$

X is singular if $4f^3 + 27g^2 = 0$

$$\Delta = \{ \text{pts w/ } | 4f^3 + 27g^2 = 0 \}$$

$$\mathbb{C} \mapsto \mathbb{C} + 1$$

$$f \in H^0(\mathcal{O}_B(-4k_B))$$

$$g \in H^0(\mathcal{O}_B(-6k_B))$$

Model of pointlike instantons:

Kodaira analyzed behavior in familiar dim $\mathbb{C}^3 = 1$

$$\Delta \subset \Sigma$$

$$\text{ord}_{\Sigma}(f), \text{ord}_{\Sigma}(g), \text{ord}_{\Sigma}(4f^3 + 27g^2)$$

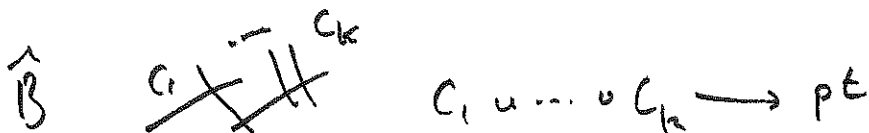
↳ surface singularity of ADE type.

$$E_{B^i} \quad 4 \quad 5 \quad 6$$

$$y^2 = x^3 + \square + t^5$$

$$y^2 = x^3 + st^5 \quad s=0 \text{ pointlike instanton}$$

$$y^2 = x^3 + f_4(s,t)x + g_6(s,t)$$



Classify all collections C_1, \dots, C_k contractible, which admit "good enough" f, g in a ubhd.

DRM-Taylor: classified C_i 's such that f and g

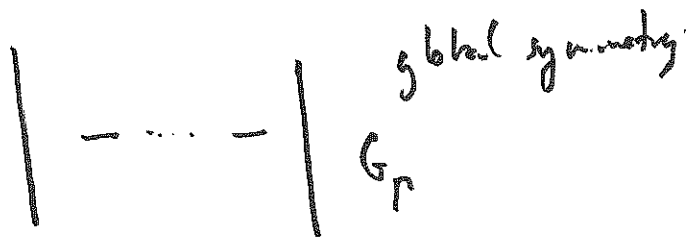
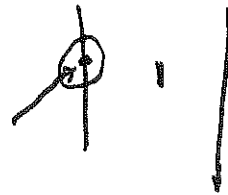
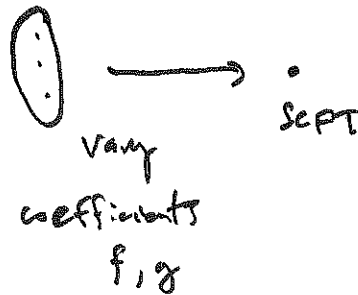
blow-down $C_1 \cup \dots \cup C_k$ to get $\mathbb{C}^2/\tilde{\Gamma}$
 where $\tilde{\Gamma} \in U(2)$

1312, ...

1502, ...

complete classification

RG flow



$\text{Hom}(\Gamma \rightarrow E_8) \leftrightarrow \text{allowed}$

