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Enumerating and exploring the sets of elliptic Calabi-Yau threefolds and fourfolds

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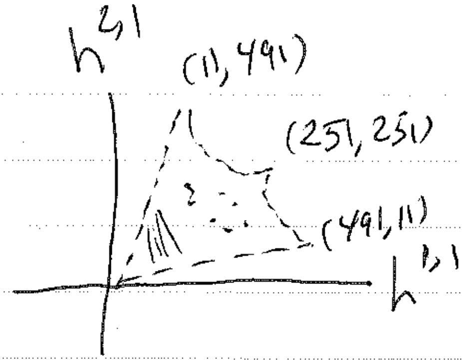
0. Intro
1. Classifying EFCY3's (start at large $h^{2,1}$)
2. Sampling EFCY4's
3. Physics:
   how does F-theory → Standard model.

§0. Classifying compact CY n-folds (K ~ 0 up to torsion)

"data" on CY 3-folds

Kreuzer-Skarke ~ 500 million

→ 30,000 Hodge number pairs



Physics of F-theory

Methodology for systematic analysis of EFCY's

a) classify Bases

$$\begin{array}{ccc} \mathbb{E} \to & X & CY3,\ CY4 \\ & \downarrow & \\ & B_{2,3} & \end{array}$$

b) "tune" Weierstrass model

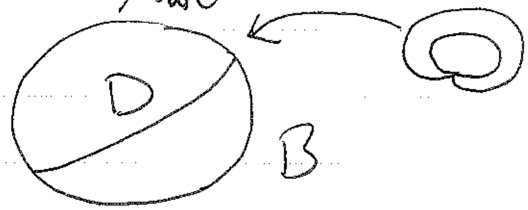
$$y^2 = x^3 + fx + g \qquad f \in \Gamma(\mathcal{O}(-4K)) \quad g \in \Gamma(\mathcal{O}(-6K))$$

$\to$ different topological types of EFCY's on B

Preview

- Systematic classification: "most" known CY's @ large h's are elliptic
- toric bases B $\to$ representative sample of known & allowed EFCY3
- structure for 4-folds is highly parallel
- modular structure from local "units"
- insight into physics

§1. Kodaira/Tate classified singularities in fibration



$$y^2 = x^3 + fx + g$$

type	$\mathrm{ord}_D f$	$\mathrm{ord}_D g$	$\mathrm{ord}_D \Delta$, $\Delta = 4f^3 + 27g^2$	Dynkin diagram / algebra
I_n	0	0	n	$\hat{A}_{n-1} \to SU(n)$
III	1	2	3	$\hat{A}_1 \to SU(2)$
$\vdots$				
II^*	4	5	10	$\hat{E}_8 \to E_8$
	4	6	12	no resolution to CY

F-theory dictionary:

Weierstrass models $\longrightarrow$ physical theories

EFCY3 $\longrightarrow$ 6D

EFCY4 $\longrightarrow$ 4D

codim 1 $\longrightarrow$ gauge group G_{NA}

codim 2 $\longrightarrow$ representation R of G_{NA} "matter"

Gross: finite # of EFCY3's up to birational isomorphism.

Plan: study B_2

minimal: $S' \quad -1 \text{ curve} \xrightarrow{\text{blow down}} S$ (repeat)

Grassi: minimal bases which support EFCY3's: $\mathbb{P}^2$, $\mathbb{F}_m$, $0 \le m \le 12$, Enriques.

Program: Construct all EFCY3's by starting with
$\mathbb{P}^2$, $\mathbb{F}_m$, $(m \le 12)$
Blowup, tune moduli in Weierstrass models.

Useful tool: "Non-Higgsable clusters"


C B_2

$C \cong \mathbb{P}^1 \qquad N(C) = \mathcal{O}(-n)$

$\Rightarrow$ forces a nontrivial singularity over C if $n > 2$.

e.g.

-12

$(K + C) \cdot C = 2g - 2 = -2.$

$C \cdot C = -12.$

$-K \cdot C = -10$

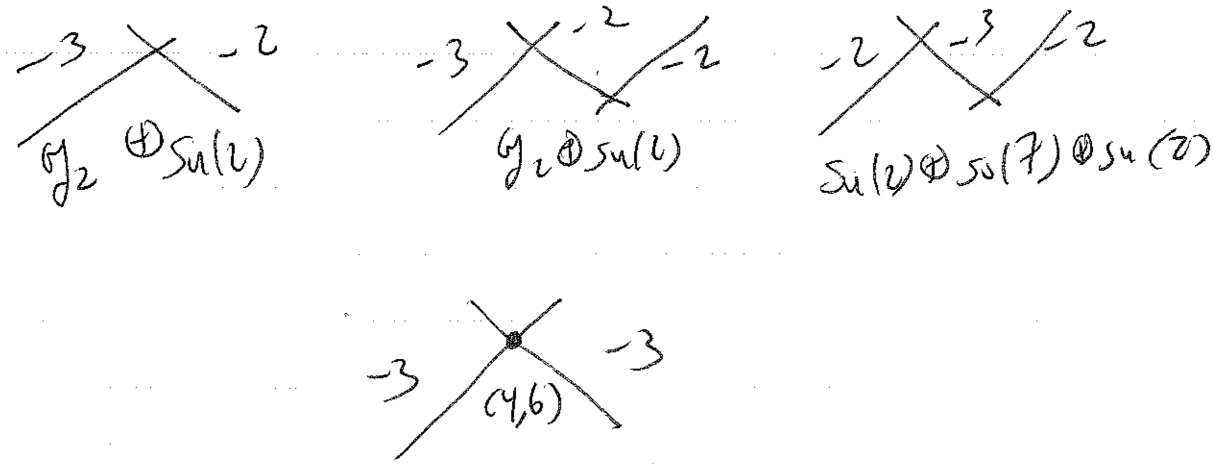
$\Rightarrow$ C rigid, C is a component of $-K$.

$-4K = 4C + X_{eff} \Rightarrow$ type II^* on C

$-6K = 5C + Y_{eff}$ (E_8 gauge group)

$\underline{-m}$, $m=3$ ... $m=12$

$su(3)$ $\quad$ E_8



Strongly restricts B_2

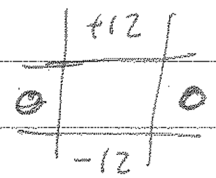
Take B_2, generic Calabi-Yau EF on B_2

$$h^{11}(X) = h^{11}(B) + r + 1, \quad r = \text{rank}(G)$$

$h^{11}(X) = h^{11}(B) + r + 1$

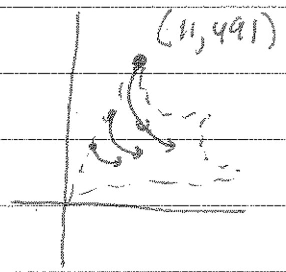
$h^{2,1} = 272 - 29(h^{11}(B) - 1) + \underset{\dim G}{V} - \underset{\dim R}{H_{ch}}$

$\mathbb{F}_{12}$



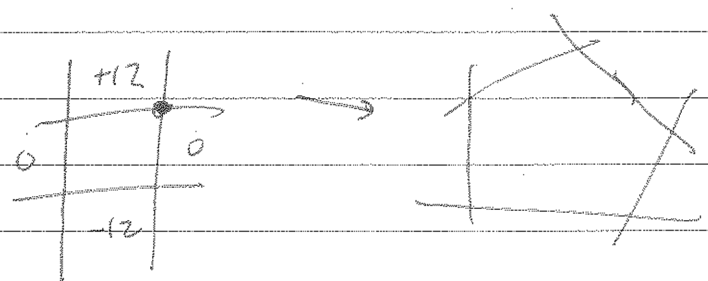
$h^{11} = 2 + 8 + 1$

$h^{21} = 272 - 29 + 248 = 491$



toric ω

$h^{21} \rightarrow h^{11}$



For toric ~ 60,000 via blow-up

$\mathbb{C}^*$-bases ~ 160,000

toric base → good sample

4-folds → Look for toric 3-fold

alt.

$$\begin{tikzcd} \mathbb{P}^2 \arrow[d] & \\ \mathbb{F}_1 \arrow[d] & \mathbb{F}_2 \\ \vdots \arrow[ur] & \end{tikzcd}$$

Start with $\mathbb{P}^3$

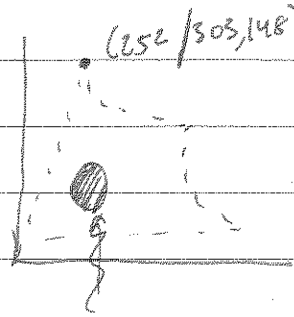
↗ Blow up at points or curves

↘ Blow down at divisors

- Monte Carlo ~~dis~~ on graph ~~the~~ generated

$\text{prob}(v_i) \propto \text{Neighbors}(v_i)$

Hodge Shield for 4-fold



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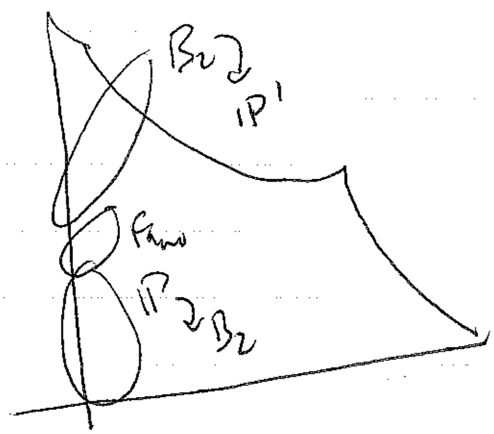
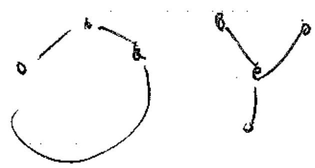


NHC for 3-fold bases:

loops, branching



only 5 products

$$SU(2) \times \begin{cases} SU(2) \\ SU(3) \\ SO(7) \\ G_2 \end{cases} \qquad SU(3) \times SO(3)$$

Monte Carlo analysis

- $N(h^{11}(B) = h)$ peaks at $\sim h^{11} = 82 \pm 6$
- total # $(= \sum N(h))$ $\sim 10^{48 \pm 2}$
- # factors in G $\sim$ linear in $h^{11}(B)$ $\sim 0.35 h^{11}(B)$
- typical group $SU(2)^{14} \times G_2^{10} \times SU(3)^2 \times SO(8) \times F_4^{3}$

- ~10% of connected pairs are $SU(3)\times SU(2)$
- divisors 6 singlets, 2 pairs, 2 big clusters
  (biggest is typically 16)

  biggest cluster found: 37 factors

(complicated topology)

§3 Physics

SM: $SU(3)\times SU(2)\times U(1)$

How do we get from F-theory?

1) "tune" on $\mathbb{B}$ ~~with no~~ without using NHC's

   e.g. $\mathbb{P}^3$, can tune $SU(5)$ "GUT"

   - cost in tuning
   - hard to tune $SU(5)$ when a lot of NHC's

   $SU(2)^{14} \leftarrow III$ $\qquad$ $SU(5) \leftarrow I_4$

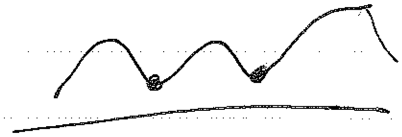
2) get $SU(3)\times SU(2)$ from NHC

   (70% have such a pair)

3) break a NHC E_8, E_7, E_6 with fluxes

$\mathcal{M}$ (EF CY4's)

Superpotential

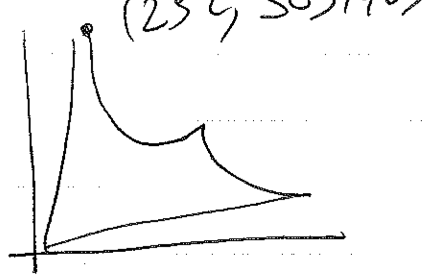


(topological fluxes.)
choice of $F \in H_4(X)$

Flux vacua:

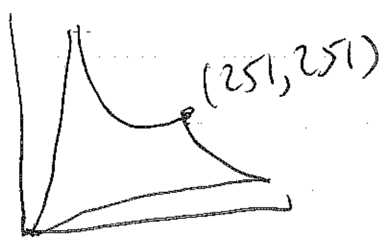
flux vacua (fixed base) $\sim 10^{h^{3,1}(X)}$

CY4



$B_2 \to \mathbb{P}^2$

CY3



N(vacua for this point M_{max}) $\sim 10^{272,000}$

Next one down is much smaller: $O(10^{-3000})$

Suggests: all but a fraction of 10^{-3000} of flux vacua come from M_{max}.

$$G = E_8^{\,9} \times F_4^{\,8} \times (G_2 \times SU(2))^{16}$$

would have to get $SU(3) \times SU(2) \times U(1)$ by flux breaking of E_8.

$\sim 1/12$ of fluxes are "turned on"

Prediction for dark matter: roughly 30 decoupled DM sectors: $G_2 \times SU(2)$
or F_4
or E_8
with some breaking by fluxes.