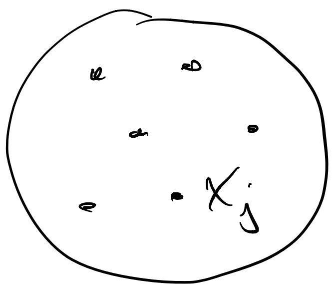
24 February 2017
A. Kitaev

On SYK model

SYK



N Majorana modes, $N \gg 1$

$$\chi_j \chi_k + \chi_k \chi_j = \delta_{jk}$$

$$\dim \mathcal{H} = 2^{N/2} \qquad (q=4)$$

$$H = \frac{1}{4!} \sum_{jklm} J_{jklm} \chi_j \chi_k \chi_l \chi_m$$

For each $j<k<l<m$, J_{jklm} is a random Gaussian variable. $\overline{J_{jklm}^2} = \dfrac{3!\, J^2}{N^3}$

$$Z = \operatorname{Tr} e^{-\beta H}$$

$$1 \ll \beta J \lesssim N$$

Imaginary time

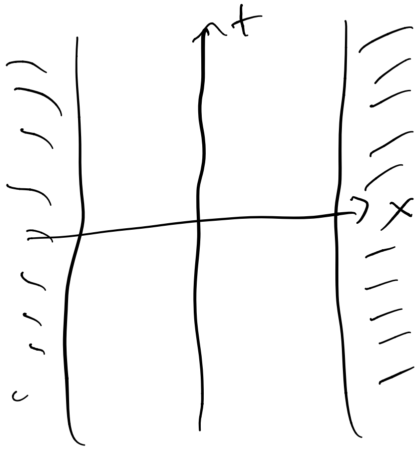
$$\beta > \tau > 0$$

$$\chi_j(\tau) = e^{\tau H} \chi_j e^{-\tau H}$$

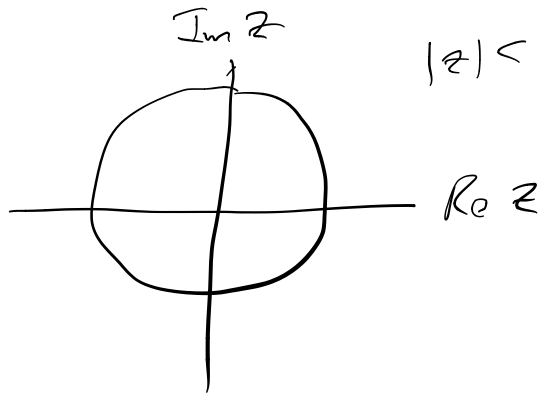
$$S = \frac{1}{16\pi G_N}\int R\sqrt{-g}\,dx + \cdots$$

$$\phi = \frac{1}{4G_N}\,; \quad S = \frac{1}{4\pi}\int \left(\phi R - U(\phi)\right)\sqrt{-g}\,dx + \cdots$$

$$U(\phi) = -2\phi \quad (+\text{ higher order terms})$$



$$I = \frac{1}{4\pi}\int_D \Big(-\phi R + \underbrace{U(\phi)}_{= -2\phi}\Big)\sqrt{g}\,dx - \frac{1}{2\pi}\int_{\partial D} K\,dl + \cdots$$



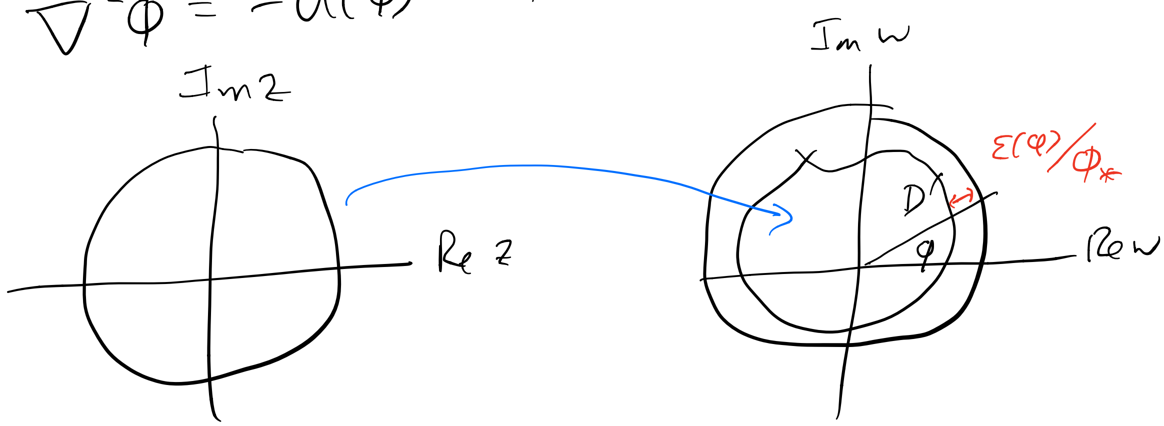
$$dl^2 = g_{\mu\nu}\,dx^\mu dx^\nu = e^{2\rho}\,dz\,d\bar{z}$$

φ: $\quad z|_{\partial D} = e^{i\varphi}$ – conformal time

$$\tau = \phi_*^{-1} \int \underbrace{dl}_{\sqrt{g_{\varphi\varphi}}\, d\varphi} \qquad \phi|_{\partial D} = \phi_*$$

$$R = U'(\phi) = -2$$

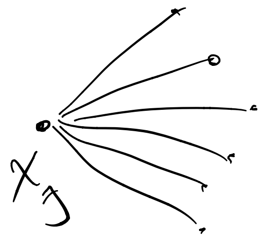
$$\nabla^2 \phi = -U(\phi) = 2\phi$$



$$dl^2 = \frac{4\, dw\, d\bar{w}}{(1 - w\bar{w})^2}$$

$$\varepsilon(\varphi) = \frac{d\varphi}{d\tau}$$

$$-\frac{1}{2\pi}\int_{\partial D} K\, dl = -\int \frac{d\varphi}{2\pi}\, \varepsilon^{-1}\left(\frac{\varepsilon^2}{2} - \frac{\varepsilon'^2}{2} + \varepsilon\varepsilon''\right)$$



$$H_j = -\mathcal{O}_j(\tau)\, \chi_j(\tau)$$

$$\mathcal{O}_j(\tau) = \frac{1}{3!} \sum_{klm} J_{jklm}\, \chi_k(\tau)\, \chi_l(\tau)\, \chi_m(\tau)$$

$\beta > \tau_1 - \tau_2 > 0$:

$$G(\tau_1, \tau_2) = \langle \chi_j(\tau_1)\, \chi_j(\tau_2) \rangle$$

$$\Sigma(\tau_1, \tau_2) = \langle \mathcal{O}_j(\tau_1)\, \mathcal{O}_j(\tau_2) \rangle$$

$H_j = -\mathcal{O}_j \chi_j$

$$\Rightarrow \left\{ \begin{array}{l} \int \left(\delta'(\tau_1 - \tau_2) + \Sigma(\tau_1, \tau_2) \right) G(\tau_2, \tau_3)\, d\tau_2 = \delta(\tau_1, \tau_3) \\ \Sigma(\tau_1, \tau_2) = J^2\, G(\tau_1, \tau_2)^{q-1} \qquad (q = 4) \\ G(\tau + 0, \tau) = \frac{1}{2} \qquad G(\tau - 0, \tau) = -\frac{1}{2} \end{array} \right.$$

$$G_\infty(\tau_1, \tau_2) = a(q)\, (\tau_1 - \tau_2)^{-2\Delta}, \quad \Delta = \frac{1}{q}$$

$\beta = \infty$

$$\Sigma_\infty(\tau_1, \tau_2) = f(q)\, (\tau_1 - \tau_2)^{-2(1-\Delta)}$$

$$\left. \begin{array}{l} G_\beta(\tau_1, \tau_2) = \tilde{a}(q) \left(\sin \dfrac{\pi(\tau_1 - \tau_2)}{\beta} \right)^{-2\Delta} \\ \Sigma_\beta(\tau_1, \tau_2) = \tilde{f}(q) \left(\sin \dfrac{\pi(\tau_1 - \tau_2)}{\beta} \right)^{-2(1-\Delta)} \end{array} \right\} \beta J \gg 1$$

$G, \Sigma, \quad \varphi : S^1 \to S^1$

$$G_\varphi(\tau_1, \tau_2) = G(\varphi(\tau_1), \varphi(\tau_2)) \, \varphi'(\tau_1)^\Delta \varphi'(\tau_2)^\Delta$$

$$\Sigma_\varphi(\tau_1, \tau_2) = \Sigma(\varphi(\tau_1), \varphi(\tau_2)) \, \varphi'(\tau_1)^{1-\Delta} \varphi'(\tau_2)^{1-\Delta}$$

$G = G_\infty, \quad \varphi(\tau) = e^{i\frac{2\pi\tau}{\beta}}$

Or, start with $G = G_\beta$.

$\varphi \circ \gamma \qquad \gamma : z \to \dfrac{az+b}{cz+d} \qquad z = e^{i\varphi}$

$\wr$

φ

Solutions are parameterized by $\mathrm{Diff}(S^1) / PSL(2,\mathbb{R})$

Effective energy

$$I_{eff} = -\frac{\alpha_S N}{\beta J} \int Sch(e^{i\varphi(\tau)}, \tau) \, d\tau$$

τ derivative

$$Sch(f(\tau), \tau) = \frac{f'''}{f'} - \frac{3}{2}\left(\frac{f''}{f'}\right)^2 \qquad \varepsilon^{-1} = \frac{d\tau}{d\varphi}$$

φ derivative

$$Sch(e^{i\varphi(\tau)}, \tau) = \frac{\varepsilon^2}{2} - \frac{(\varepsilon')^2}{2} + \varepsilon\varepsilon''$$

$$G(\tau_1,\tau_2) = (\tau_1-\tau_2)^{-2\Delta} + u(\tau)\,(\tau_1-\tau_2)^{-2\Delta+2}$$

$\tau_1-\tau_2 \to 0$

$\tau_1, \tau_2 \to \tau$

$$G_\infty = (\tau_1-\tau_2)^{-2\Delta}$$

$$= \left(\frac{f(\tau_1)-f(\tau_2)}{\sqrt{f'(\tau_1)f'(\tau_2)}}\right)^{-2\Delta}$$

$$f(\tau) = e^{-i\varphi(\tau)}$$

$$\frac{f(\tau_1)-f(\tau_2)}{\sqrt{f'(\tau_1)f'(\tau_2)}} \simeq (\tau_1-\tau_2)\left(1+\frac{1}{12}\operatorname{Sch}(f(\tau),\tau)\cdot(\tau_1-\tau_2)^2\right)$$

$$u(\tau) = -\frac{\Delta}{6}\operatorname{Sch}(f(\tau),\tau)$$