# Mirror Symmetry & Quintic 3-fold

$V = \{ f_5(x_1, \dots, x_5) = 0 \} \subset \mathbb{P}^4$
generic

$h^{1,1}(V) = 1 \qquad h^{2,1} = 101$

$K_V \equiv 0 \quad \sim \quad \mathrm{Res.}\left\{ \frac{\omega}{f_5(x_1,\dots,x_5)} \right\}\Big|_V$ is a hol. 3-form on $V$

$\omega$ is standard 4-form on $\mathbb{P}^4$

$V$ is (first) example of a Calabi-Yau 3-fold

Type IIA string theory on $V \times \mathbb{R}^{1,3}$ $\sim$ Type IIB on $V \times \mathbb{R}^{1,3}$

$h^{1,1}(V) =$ dim. of "vector" moduli $\quad$ = dim of "hyper" moduli

$h^{2,1}(V) =$ dim of "hyper" moduli $\quad$ = dim of "vector" moduli

Mirror Symmetry: Started as a guess ~ what if $\exists$ CY $W$ s.t.
IIA on $V \times \mathbb{R}^{1,3}$ is the same as IIB on $W \times \mathbb{R}^{1,3}$ &
IIB on $V \times \mathbb{R}^{1,3}$ is the same as IIA on $W \times \mathbb{R}^{1,3}$

# Greene-Plesser:

Described mirror of a hypersurface of Fermat type;

$$V = \{X_1^{a_1} + X_2^{a_2} + \dots + X_5^{a_5} = 0\} \subseteq \mathbb{P}_{\mathbb{Q}}^{(\frac{m}{a_1}, \dots, \frac{m}{a_5})}$$

($a_i$, $m$ chosen so $V$ is a possibly singular CY hypersurface)

$$V_\psi = \{x_1^5 + \dots + x_5^5 + \psi x_1 x_2 x_3 x_4 x_5 = 0\}$$

crepant res of

$$\widetilde{V_\psi / \mathbb{Z}_5^3}, \text{ where } \mathbb{Z}_5^3 = \{(\zeta_1, \dots, \zeta_5) \in \mu_5^5 \mid \textstyle\prod \zeta_i = 1\} / (\zeta,\zeta,\zeta,\zeta,\zeta)$$

Then $h^{1,1}(V) = 1$ & $h^{2,1}(V) = 101$

$$h^{1,1}(\widetilde{V_\psi / \Gamma}) = 101 \quad h^{2,1}(\widetilde{V_\psi / \Gamma}) = 1$$

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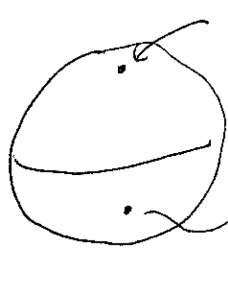
Idea: $h^{2,1}(\widetilde{V_\psi/\Gamma}) = 1$

$\psi^{-5} = \sum \mathcal{O} \psi^{-5a}$

Moduli of $\psi \to$



conifold pt ($\psi = 0$)
(Fermat)

$\psi = \infty$ even more singular

$\sum c_a t^a \sim$ "$c_a$ = # of rational curves on $V$ of degree $a$"

↑

Quantum Corrections

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At the time: (what was known)

If $a=1$ 2875

$a=2$ 609,250

$a=3$ Ellingsrud - Strømme

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Physics prediction

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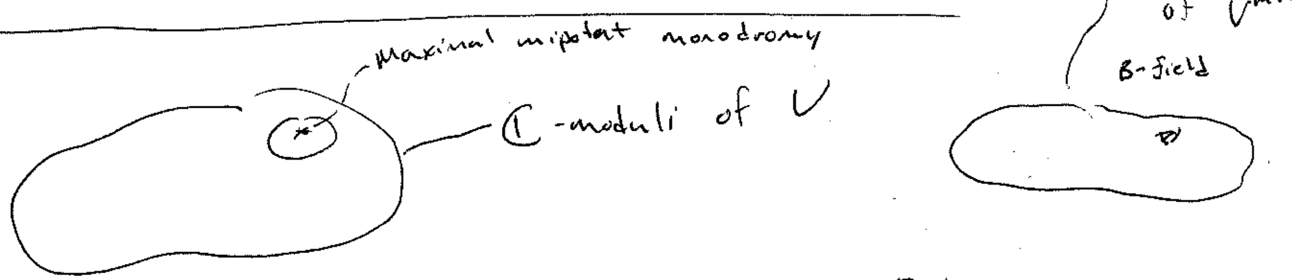


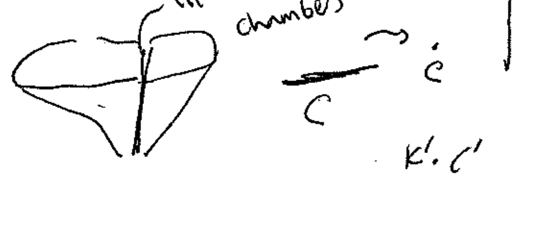

Greene-Plesser

Extended to toric varieties by Batyrev

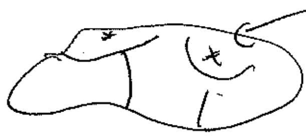
Givental, Lian, Yau — Mirror Theorem

Kähler cone $\subseteq$ ~~$\mathbb{R}^{h^{1,1}}$~~ $\sim$ complexify by adding $i\mathbb{R}^{h^{1,1}}$

$\subseteq H^2(V,\mathbb{R})$



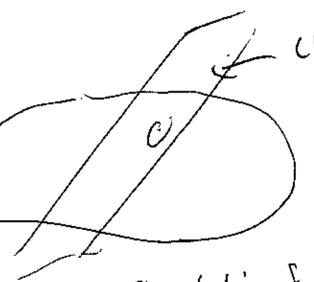
# Moveable cone $\subseteq H^2(V,\mathbb{R})$




Recently:

New way of evaluating correlation functions for the quintic mirror

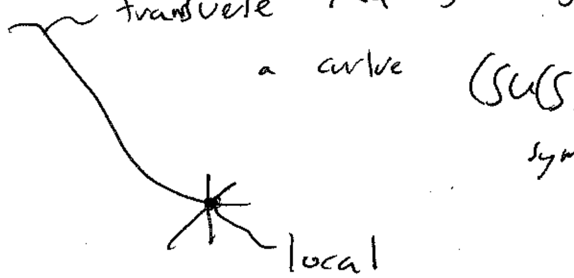
$\Rightarrow$ Predictions about curve classes




Mori Cone of quintic Mirror $=$ dual of Kähler Cone

# Mirror Quintic

$V_\psi/\mathbb{Z}_5^3$ has

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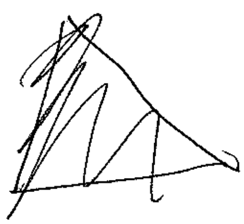
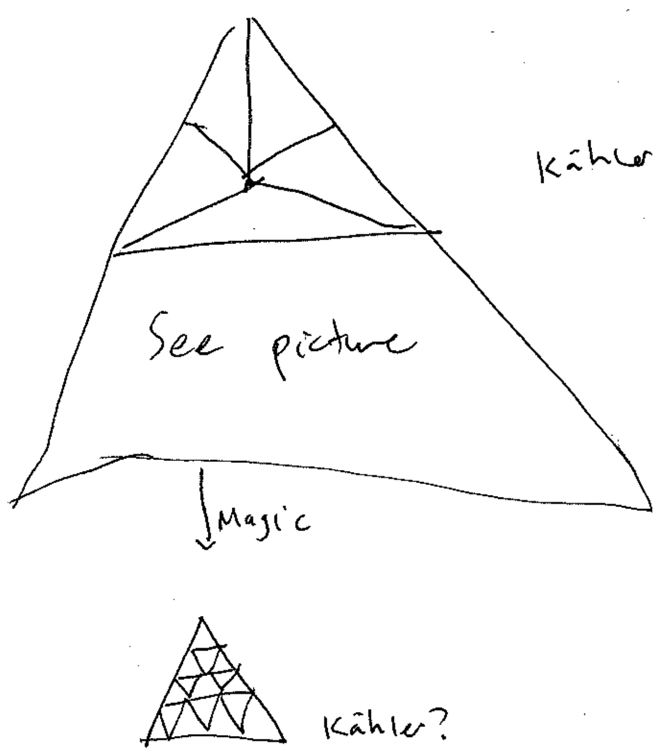
Algorithm for crep. resolutions:

- Blow up intersections
- Blow up singular curves (twice each)
- Find a small resolution of 60 singular points created in Step 1.





$$t_1^5 + \cdots + t_5^5 - 5\psi t_1 \cdots t_5 = 0$$

$$\mathbb{Z}_5^4 = \operatorname{Ker}\left(\mathbb{Z}_5^5 \xrightarrow[\phi]{} \mathbb{Z}_5^5\right)$$

$$(x_1, \ldots, x_5) \mapsto x_1 \cdot x_2 \cdots x_5$$

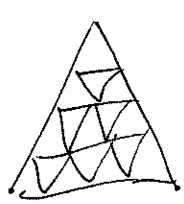
$u = t_1 t_2 t_3 t_4 t_5 \qquad v = t_1^5 \quad x = t_3^5 \quad z = t_5^5$

$\qquad\qquad\qquad\qquad\qquad w = t_2^5 \quad y = t_4^5$

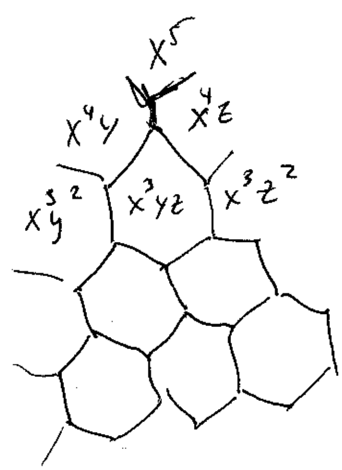
$$\Rightarrow v + w + x + y + z = 5\psi u$$

$$vwxyz = u^5$$

$\mathbb{P}^4 / \mathbb{Z}_5^3$ is a $(5,1)$ complete intersection in $\mathbb{P}^6$



Dual



$D_m$ = divisor (labelled by monomial $m$)

$D_m \cap D_n = \gamma_{m,n}$

$s \in \{v, w, x, y, z\}$

$l_s$ is a line in $D_s^s \cong \mathbb{P}^2$

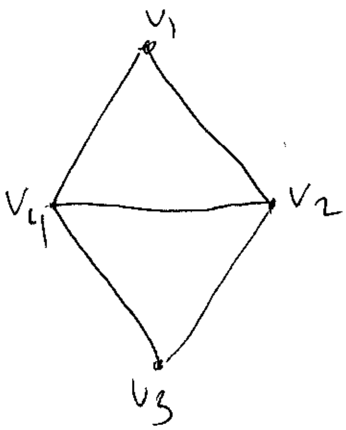
$\mathbb{P}$

Claim

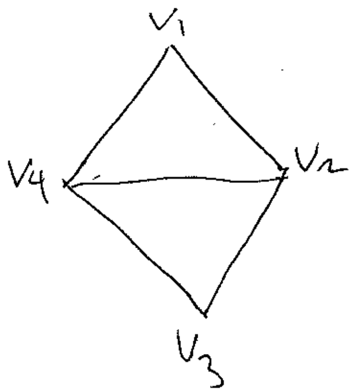
Mori cone of quintic mirror is spanned by those curves

Good triangulation ⇒ in toric geometry

2-forms on large triangles induce Kahler form on triangulated sing, which correspond to blow-up of original




$v_1 + v_3 \leq v_2 + v_4$




$V_1 + V_3 \geq V_2 + V_4$

blow up intersection pts

ʌp up Singular Curves (twice each)
for
curvature

nd Small res, of 60 Sing pts Created