



## GEOMETRY, TOPOLOGY, AND PHYSICS SEMINAR

**Manifold Pairings and  
Three-Manifold Positivity****Mike Freedman (Microsoft Research)**Friday, October 27, 2006, 3:30 p.m.  
Room 6635 South Hall**Abstract:**

Gluing two manifolds along a common boundary so that the boundary disappears is a fundamental operation of topology. In the spirit of physics we may consider “superpositions”, i.e., complex linear combinations, of  $n$ -manifolds bounding a fixed  $(n-1)$ -manifold  $S$ . Two such superpositions may be glued along  $S$  to yield a superposition of closed  $n$ -manifolds. It is known (work with: Wang, Slingerland, Kitaev, and Walker and another study by Teichner and Kreck)) that in dimensions  $n > 3$  these pairings can have null vectors,  $v \neq 0$  yet  $\langle v, v \rangle = 0$ . Joint work with Calegari and Walker shows that when  $n = 3$  these pairings are positive (lack null vectors). The proof is fun because it uses nearly everything we know about three manifolds. In historical order: loop theorem/Dehn’s lemma, prime decomposition, JSJ decomposition, Geometrization of Haken manifolds, residual finiteness of 3-manifold groups, the unitarity of finite group TQFTs, and crucial Perelman lemmas on Ricci flow/surgery (after Agol et al.). The original motivation for asking about “positivity” comes from the study of two dimensional electron gasses (FQHE) and other two dimensional systems that might admit topological phases.

Information about future meetings of this seminar can be found at  
<http://www.math.ucsb.edu/~drm/GTPseminar/>