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Geometry, Topology, and Physics Seminar

Embeddability of simplicial complexes is undecidable

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Friday, January 10, 2020, 4:00 p.m.
Room 6635 South Hall

Abstract: We consider the following decision problem EMBED(k,d) in computational topology (where $k \leq d$ are fixed positive integers): Given a finite simplicial complex K of dimension k, does there exist a (piecewise-linear) embedding of K into $\mathbb{R}^d$?

The special case EMBED(1,2) is graph planarity, which is decidable in linear time, by the well-known algorithm of Hopcroft and Tarjan.

In higher dimensions, EMBED(2,3) and EMBED(3,3) are known to be decidable (as well as NP-hard), and recent results of Čadek et al., in combination with a classical theorem of Haefliger and Weber, imply that EMBED(k,d) can be solved in polynomial time for any fixed pair (k,d) of dimensions in the so-called *metastable range* $d \geq (3(k+1))/2$.

Here, by contrast, we prove that EMBED(k,d) is algorithmically undecidable for almost all pairs of dimensions outside the metastable range, namely for $8 \leq d < \lfloor (3(k+1))/2 \rfloor$. This almost completely resolves the decidability vs. undecidability of EMBED(k,d) in higher dimensions and establishes a sharp dichotomy between polynomial-time solvability and undecidability.

Our proof builds on work by Čadek et al., who showed how to encode an arbitrary system of Diophantine equations into a homotopy-theoretic extension problem. We turn their construction into an embeddability problem, using techniques from piecewise-linear (PL) topology due to Zeeman, Irwin, and others.

Information about future meetings of this seminar can be found at
http://web.math.ucsb.edu/~drm/GTPseminar/