Symmetry in QFT and Gravity

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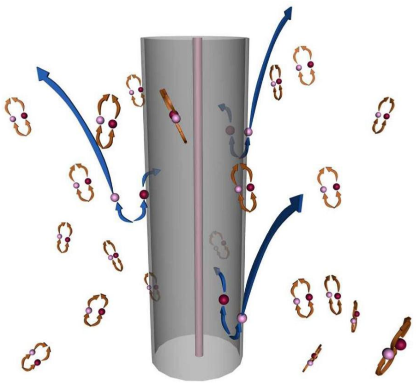
Based on:

1-4 : arXiv:1810.05337, arXiv:1810.05338
5 : arXiv:2109.03838

with Daniel Harlow

Quantum gravity does not have exact global symmetry.

Standard argument:



Suppose there is a global symmetry G.

We can combine a large number of charged matters to make a **black hole in an arbitrary large representation** of G.

Let it Hawking-radiate, keeping its mass >> the Planck mass.

Since the Hawking radiation is G-blind, the black hole still contains the large representation of G with the number of states **exceeding the Bekenstein-Hawking entropy** formula.

This argument has loopholes, and
it does not exclude discrete global symmetry.

1. Global Symmetry in Quantum Field Theory

QFT on $\mathbb{R} \times \Sigma$ has global symmetry G. $\Longleftrightarrow$

(a) $\exists\, U(g, \Sigma)$: unitary homomorphism $G \to \mathcal{U}(\mathcal{H}_\Sigma)$

$\mathcal{U}(\mathcal{H})$: set of unitary operators on $\mathcal{H}$.

(b) $\forall\, S \subset \Sigma$, $U(g; \Sigma)^\dagger \mathcal{A}[S]\, U(g, \Sigma) = \mathcal{A}[S]$.

$\mathcal{A}[S]$: algebra of operators localized in S.

(c) $U(g, \Sigma)$: faithfully represents G on $\mathcal{A}[S]$.

(d) $U(g, \Sigma)$: preserves $T_{\mu\nu}(x)$, i.e. topological.

SPLITTABILITY

If G is continuous and has a Noether current J_a,

$$U(g, \Sigma) = \exp\left(i\,\varepsilon^a \int_\Sigma * J_a\right) \quad \text{for} \quad g = e^{i \varepsilon^a T_a}$$

then, we can define $U(g, S)$ for $\forall$ open $S \subset \Sigma$, which acts as $U(g, \Sigma)$ on $\mathcal{A}[S]$ and commutes with $\mathcal{A}[\mathrm{int}(\Sigma \setminus S)]$.

$$U(g, \bigcup_i S_i) = \prod_i U(g, S_i) \quad \text{for disjoint } S_i\text{'s.}$$

This splittability holds for any symmetry (including discrete symmetry) if QFT has the split property.

[Buchholz, Doplicher, Longo, Ann Phys, 170 (1986) 1.]

Since the split property is important for holography and entanglement, let me elaborate a little ...

QFT on $\mathbb{R} \times \Sigma$ has the split property

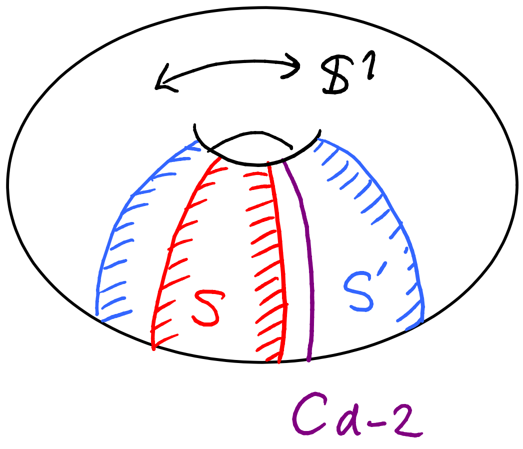
$\Leftrightarrow$ $\forall$ open $S, S' \subset \Sigma$ s.t. closure$(S) \subset$ int(S'),

$\exists$ type I factor $\mathcal{N}$, s.t. $\mathcal{A}[S] \subset \mathcal{N} \subset \mathcal{A}[S']$.

Comments :

- von Neumann algebra $M \subset \mathcal{L}(\mathcal{H})$ is a factor iff its center is trivial (i.e. contains λI, $\lambda \in \mathbb{C}$ only).
- A factor is type I iff it contains a minimal projection.
- If M is a type I factor, $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$ such that
  $M = \mathcal{L}(\mathcal{H}_A) \otimes I_{\bar{A}}$, (Commutant of M) $= I_A \otimes \mathcal{L}(\mathcal{H}_{\bar{A}})$.
- $\mathcal{A}[S]$ is not expected to be a type I factor.
- QFT on $\mathbb{R}^d$ is expected to have the split property.

Pure Maxwell theory on $\Sigma = S^1 \times C_{d-2}$ does not split.



$$\Phi \equiv \int_{C_{d-2}} {}^*F$$ electric flux on C_{d-2}

commutes with $\mathcal{A}[S']$.

$\Rightarrow \Phi \in$ center of $\mathcal{N}$.

But, Φ is nontrivial on $\mathcal{L}(\mathcal{H})$

(does not commute with Wilson lines on S^1).

Comments

- The trouble is due to an exact one-form symmetry.
- If the gauge group is compact $U(1)$, the split property can be restored by adding heavy charges in UV.
- Relatedly, there is no gauge invariant noether current for $U(1)$ global symmetry in pure $\mathbb{R} \times \mathbb{R}$ gauge theory.

Recap: QFT on $\mathbb{R} \times \Sigma$ splits iff $\mathcal{A}[R] \subset \mathcal{N} \subset \mathcal{A}[R']$.

type I factor

If $\mathcal{N}$ is type I factor,

$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$ and $\mathcal{N} = \mathcal{L}(\mathcal{H}_A) \otimes I_{\bar{A}}$.

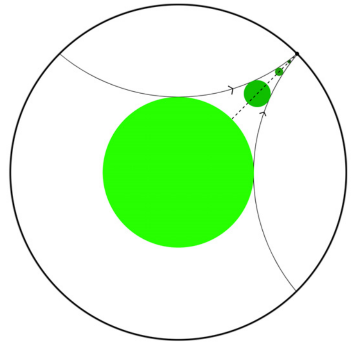
- QFT on $\Sigma = \mathbb{R}^{d-1}$ is expected to split.

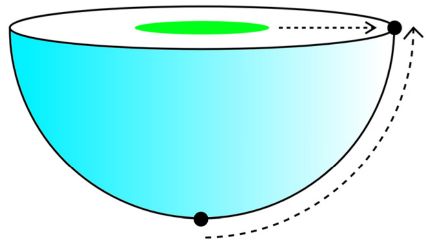
  The pure Maxwell theory does not split on $\Sigma = \mathbb{S}^1 \times C_{d-2}$.

- CFT on $\Sigma = \mathbb{S}^{d-1}$ is also expected to split by the state-operator correspondence.

  --- This is sufficient for our purpose.

2. Global Symmetry in Quantum Gravity in AdS





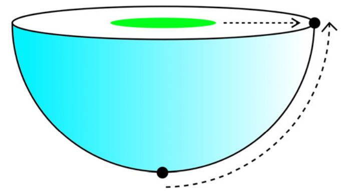
By requiring that global symmetry in the bulk is defined in such a way that it reduces to that in QFT in the limit of $G_N \to 0$, and with symmetry action on local operators in CFT implied by that on quasi-local operators with gauge and gravitational dressings in the bulk,

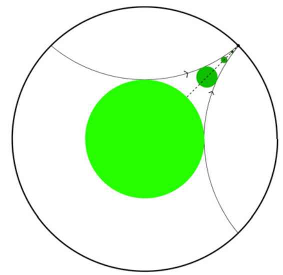
it follows

that global symmetry in gravity in AdS is global symmetry in its CFT dual.

But, there are more conditions.

Global symmetry of quantum gravity in AdS





(a) is global symmetry of its dual CFT

(b) acts locally within quasi-local bulk operators, preserving their gravitational dressings.

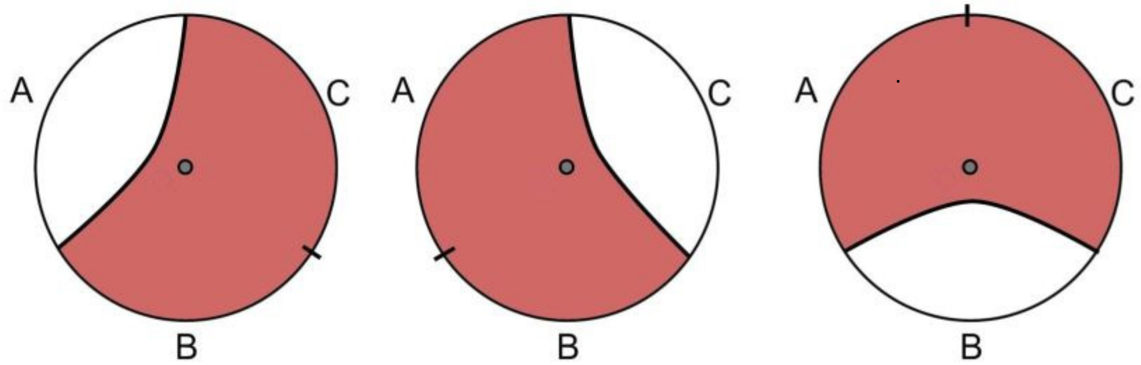
(c) acts faithfully on gauge invariant quasi-local bulk operators.

(d) acts distinctly from asymptotic conformal symmetry

(For example, fermion parity is 2π rotation and does not count.)

3. No Global Symmetry in Quantum Gravity in AdS

We use the entanglement wedge reconstruction.



Local excitation deeper in the AdS bulk requires acting on entanglement properties of the vacuum over a larger region in CFT.

If a gravitational theory in AdS has global symmetry, there must be a quasi-local bulk operator that transforms faithfully under the symmetry.

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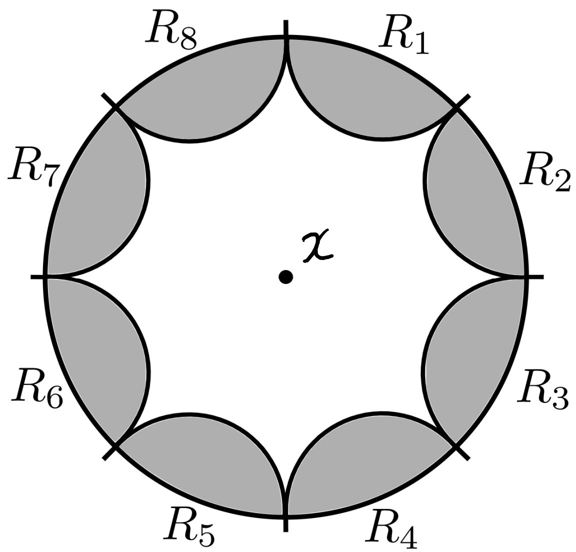


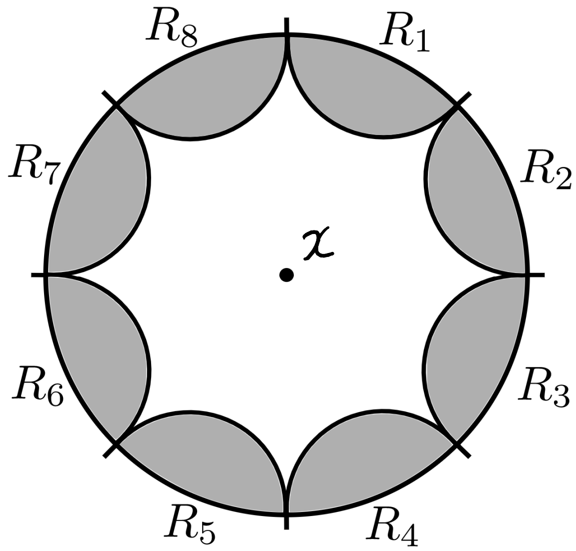
Symmetry generator

$$U(g,\Sigma) = \prod_i U(g, R_i)$$

commutes with a quasi-local operator at x.

Contradiction

Harlow + HO:
arXiv:1810.05337 (5 page summary, Phys. Rev. Lett.)
arXiv:1810.05338 (175 page complete proof, CMP)



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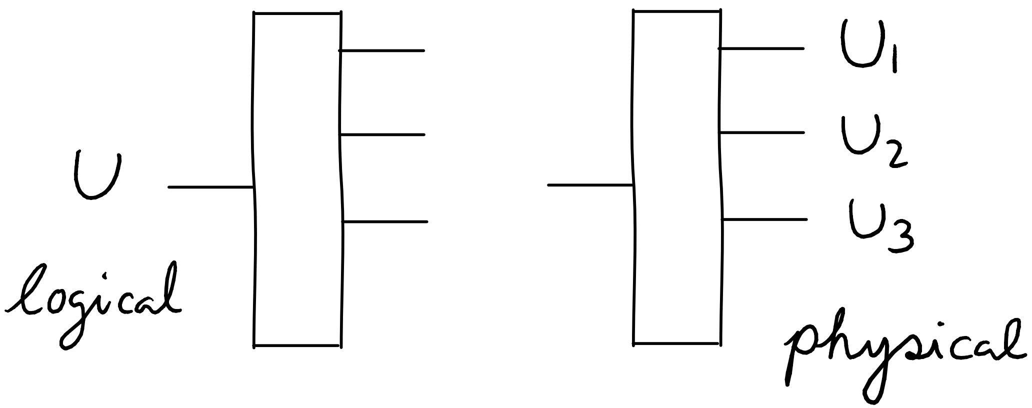
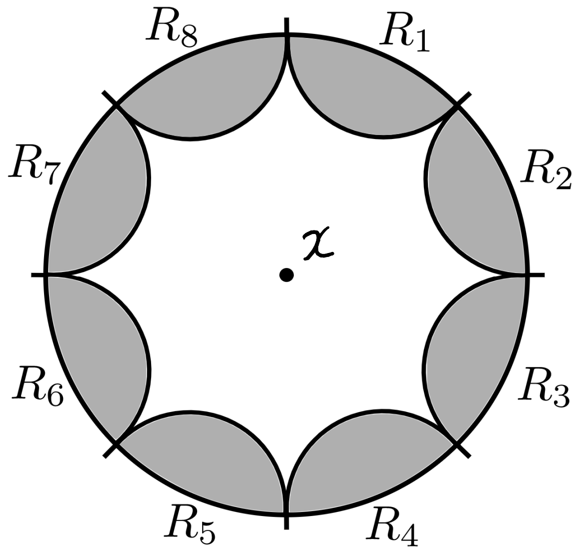


Nontrivial U on logical qubits cannot be expressed as $\bigotimes_i U_i$ on physical qubits if each U_i preserves the code subspace.



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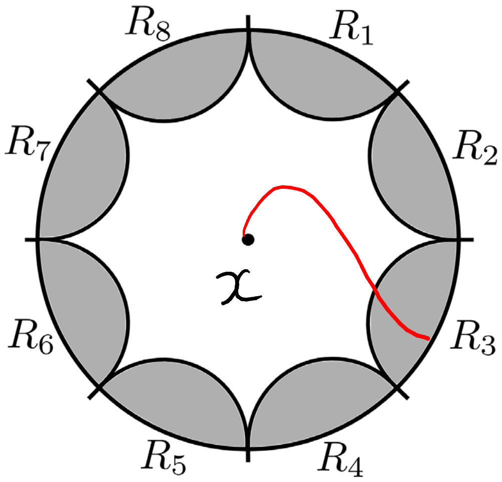
commutes with a quasi-local operator at x.

- The argument works for spontaneously broken global symmetry. For example, shift symmetry $\phi \to \phi + \text{const}$ for a scalar field must be broken.
- The argument also works for discrete spacetime symmetry such as P and T. They are gauge symmetries and quantum gravity must sum over non-orientable manifolds.
- The argument does not work for 2d gravity (e.g. on string worldsheet) and 3d oriented pure Einstein gravity.

4. Completeness of Gauge Representations



Global symmetry G in CFT corresponds to long-range gauge symmetry G in AdS.

Are they the same G ?

We need to show $U(g, \Sigma)$ acts faithfully on bulk operators.

Example : W_α , Wilson line in representation α through AdS wormhole.

W_α acts on $\mathcal{H}_{CFT_L} \otimes \mathcal{H}_{CFT_R}$

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W_α acts on $\mathcal{H}_{CFT_L} \otimes \mathcal{H}_{CFT_R}$

$$U(g, \Sigma_L)^\dagger \, W_\alpha \, U(g, \Sigma_L) = \mathcal{D}_\alpha(g) \, W_\alpha$$

By the Peter-Weyl theorem, $\forall g$, $\exists \alpha$ s.t.

$$U(g, \Sigma_L)^\dagger \, W_\alpha \, U(g, \Sigma_L) \, W_\alpha^\dagger = \mathcal{D}_\alpha(g) \neq \mathbb{1}$$

This would not happen if $\exists g_0$ s.t. $U(g_0, \Sigma)$ commutes with any bulk operators (i.e. not faithful)

$U(g, \Sigma)$ acts faithfully on bulk operators.

If the symmetry group G is compact, one can find a finite dimensional faithful representation ρ.

$$\Downarrow$$

$\rho^{n} \otimes \bar{\rho}^{m}$ generate all finite dimensional irreducible representation of G.

This proves the completeness hypothesis.

5. Asymptotic Density decomposed into Representations

Harlow + HO: arXiv:2109.03838

If a holographic CFT is invariant under a **finite group** G, we can use the AdS gravity to show:

$$\mathrm{Tr}\left[U(g)\, e^{-\beta H}\right] \simeq \delta(g)\, \mathrm{Tr}\left[e^{-\beta H}\right] \quad \text{for large } T = \frac{1}{\beta}$$

over the CFT Hilbert space

character of α.

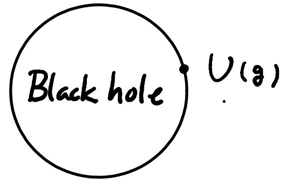
Since $\displaystyle \delta(g) = \sum_{\alpha:\,\text{irrep's}} \frac{\dim \alpha}{|G|}\, \chi_\alpha(g)$,

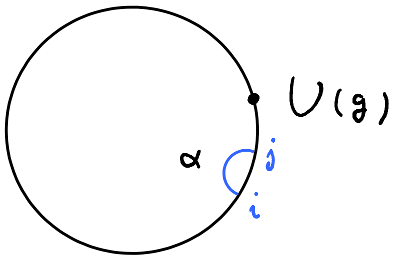
each representation α occupies $\dfrac{(\dim \alpha)^2}{|G|}$ of all states.

$$\left(\text{Note: } \sum_\alpha \frac{(\dim \alpha)^2}{|G|} = 1 \right)$$

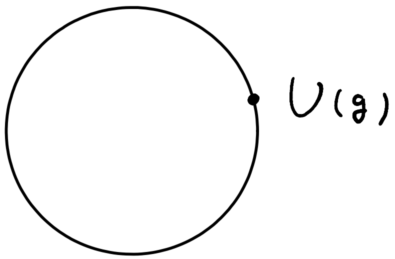
Using the AdS gravity to derive $\mathrm{Tr}[U(g)\,e^{-\beta H}] \simeq \delta(g)\,\mathrm{Tr}[e^{-\beta H}]$

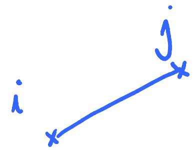
For large $T = 1/\beta$,
(above the Hawking-Page transition) $\mathrm{Tr}[U(g)\,e^{-\beta H}] =$





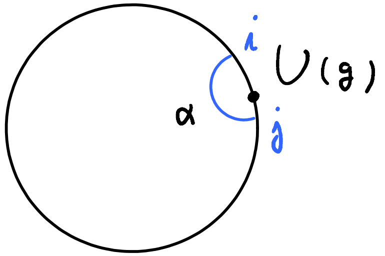
$\propto \delta_i^{\ j}$



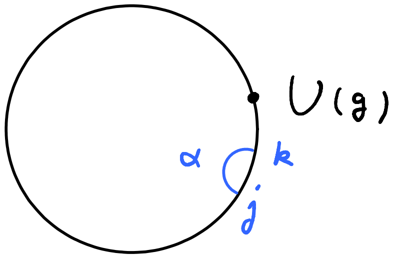


= Wilson line with $i, j \in \alpha$.

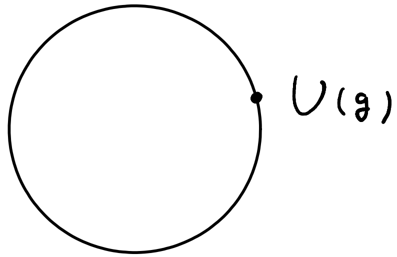
$\|$



$= D_\alpha(g)_i^{\ k}$



$\propto D_\alpha(g)_i^{\ j}$



$\Rightarrow \left(\delta_i^{\ j} - D_\alpha(g)_i^{\ j}\right)\mathrm{Tr}[U(g)\,e^{-\beta H}] = 0$ for any α.

$$\mathrm{Tr}\left[U(g)\, e^{-\beta H} \right] \simeq \delta(g)\, \mathrm{Tr}\left[e^{-\beta H} \right] \quad \text{for large } T = \frac{1}{\beta}$$

The derivation does not use the gravity action or any other dynamical information, except for the existence of a holographic gravity dual, where the thermal circle contracts in high temperature.

This may hold for a larger class of QFT's

e.g. True for any unitary CFT in 2 dimensions

Pal + Sun : 2004.12557

Perhaps, the completeness is sufficient to show this

ρ : faithful representation

$\Rightarrow \rho_n = (1 \oplus \rho \oplus \rho^+)^n$, $n = 1, 2, \cdots$ generate all rep's.

How many copies of α-rep are there in ρ_n ?

$$N_\alpha^{(n)} = \frac{1}{|G|} \sum_{g \in G} \chi_\alpha(g)\, \chi_{\rho_n}(g)$$

One can show :

$$\lim_{n \to \infty} \frac{N_\alpha^{(n)}}{N_0^{(n)}} = \dim \alpha$$

$$\mathrm{Tr}\left[\, U(g)\, e^{-\beta H} \right] \simeq \delta(g)\, \mathrm{Tr}\left[e^{-\beta H} \right] \quad \text{for large } T = \frac{1}{\beta}$$

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- Derivable assuming a certain property of the thermo-field double state

  Magán : 2111.02418 using Casini, Huerta, Magán, Pontello : 2008.11748

- Shown for weakly coupled QFT's. Cao, Melia, Pal : 2111.04725

  Generalization to compact Lie groups. Lee, Kang + H.O. to appear

Questions for the Future:

- Any global symmetry in low energy effective theory of quantum gravity must be approximate.

  How is it broken?

  Are symmetry breaking terms
  power suppressed by the Plank mass?

- Can we find an upper bound on the mass of the charged particles required by the completeness hypothesis?

- Beyond AdS/CFT, more general background.