Celestial Holography
from Bottom-up to Top-down

The Western Hemisphere Colloquium on
Geometry and Physics

Sabrina Pasterski
Perimeter Institute
12/11/23
**Soft Thm=Ward Id**
- u-falloffs and antipodal matching

**Celestial OPE**
- analytic structure of amplitudes, inner product

**Boost Basis**
- principal series vs highest weight reps

**Chiral Algebra**
- twistor & twisted holography re-interpretations
WHAT IS CELESTIAL HOLOGRAPHY?
bh info + string th \rightarrow holographic principle
$\Lambda = 0$ \quad \text{vs} \quad \Lambda \to 0$
Lesson 1: BMS >> Poincare: $\Lambda = 0$ spacetimes have a much larger class of possible symmetries.

$$\text{ASG} = \frac{\text{allowed gauge symmetries}}{\text{trivial gauge symmetries}}$$
\[ ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \]

\[ G_{\mu\nu} = 8\pi G T_{\mu\nu} \]
$u = t - r$

$v = t + r$
In Bondi gauge the metric near future null infinity takes the form

\[ ds^2 = -du^2 - 2dudr + 2r^2 \gamma_{z\bar{z}} dzd\bar{z} + 2 \frac{m_B}{r} du^2 \\
+ (r C_{zz} dz^2 + D^z C_{zz} dudz + \frac{1}{r} \left( \frac{4}{3} N_z - \frac{1}{4} \partial_z (C_{zz} C^{zz}) \right) dudz + c.c.) + ... \]

which is preserved by the residual diffeomorphisms

\[ \xi^+ = (1 + \frac{u}{2r}) Y^{+z} \partial_z - \frac{u}{2r} D^z D_z Y^{+z} \partial_z - \frac{1}{2} (u + r) D_z Y^{+z} \partial_r + \frac{u}{2} D_z Y^{+z} \partial_u + c.c. \\
+ f^+ \partial_u - \frac{1}{r} (D^z f^+ \partial_z + D^z f^+ \partial_r) + D^z D_z f^+ \partial_r \]

Radiative Data

Superrotations

Supertranslations
Form this bulk analysis we land on the BMS group

\[
[L_m, L_n] = (m - n)L_{m+n}, \quad [\tilde{L}_m, \tilde{L}_n] = (m - n)\tilde{L}_{m+n},
\]

\[
[L_n, P_{k,l}] = (\frac{1}{2}n - k)P_{k+n,l}, \quad [\tilde{L}_n, P_{k,l}] = (\frac{1}{2}n - l)P_{k,l+n},
\]

\[
[P_{m,n}, P_{k,l}] = 0.
\]

where \(n, m \in \{-1, 0, 1\}\) & \(k, l \in \left\{-\frac{1}{2}, \frac{1}{2}\right\}\) give the Poincare subalgebra
We can see this enhancement from the boundary perspective

\[ ds^2 = -c^2 du^2 + dz d\bar{z} \]

as a Carrollian limit of a Lorentzian CFT\(_3\).

\[ \lim_{c \to 0} [\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = \alpha g_{\mu\nu}] \]

\[ \lim_{c \to 0} c^2 [\nabla^\mu \xi^\nu + \nabla^\nu \xi^\mu = -\alpha g^{\mu\nu}] \]
\[ ds^2 = -c^2 dt^2 + d\vec{x}^2 \]

- \( c \to \infty \) Galilean
- \( c = 1 \) Lorentzian
- \( c \to 0 \) Carrollian
$\Lambda < 0$

$\Lambda = 0$
\[ \Lambda < 0 \quad \text{and} \quad \Lambda = 0 \]

\[ SO(2,3) \quad \text{and} \quad ISO(1,3) \]

\[ AdS_4/CFT_3 \quad \text{and} \quad Mink_4/CCFT \]

\[ \Lambda \propto c \to 0 \]

Celestial CFT_2

Carrollian CFT_3
Lesson 2: These are indeed symmetries of the perturbative S-matrix!

\[ \langle \text{out} | Q^+ [Y] S - SQ^- [Y] | \text{in} \rangle = 0 \]
The free data for our solution takes the form

\[ \{ m_B(u_0, z, \bar{z}), N_z(u_0, z, \bar{z}), C_{zz}(u, z, \bar{z}) \} \]

where the \( u \) dependence of the Bondi mass and angular momentum aspect are fixed by the constraint equations

\[ n^\mu [G_{\mu\nu} - 8\pi G T_{\mu\nu}] = 0 \]
The free data for our solution takes the form

$$\{m_B(u_0, z, \bar{z}), N_z(u_0, z, \bar{z}), C_{zz}(u, z, \bar{z})\}$$

where the $u$ dependence of the Bondi mass and angular momentum aspect are fixed by the constraint equations

$$\partial_u m_B = \frac{1}{4} [D_z^2 N^{zz} + D_{\bar{z}}^2 N^{\bar{z}\bar{z}}] - T_{uu}$$
1. $Q^+ = Q^-$ due to antipodal matching of $m_B$ & $N_z$ across $i^0$

2. Integration by parts turns the charges into fluxes

$$Q_f^+ = \frac{1}{8\pi G} \int_{\mathcal{J}^+} 2m_B f$$

$$8\pi G Q^+[Y] = \int_{\mathcal{I}^+} \sqrt{g} d^2z d\mu \left[ -\frac{1}{2} D_z^2 Y^z u \partial_u C_{zz} + Y^z T_{uz} + u D_z Y^z T_{uu} + h.c. \right]$$

$$Q^+[Y] = Q^+_S[Y] + Q^+_H[Y]$$

$$C_{zz}|_{\mathcal{J}^+} = C_{zz}|_{\mathcal{J}^-}, \quad m_B|_{\mathcal{J}^+} = m_B|_{\mathcal{J}^-},$$
\[ h_{\mu \nu}(x) = \sum_{\alpha = \pm} \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_q} \left[ \epsilon^{\alpha*}(\vec{q})a_{\alpha}(\vec{q})e^{i\vec{q} \cdot \vec{x}} + \epsilon_{\mu \nu}(\vec{q})a_{\alpha}(\vec{q})^\dagger e^{-i\vec{q} \cdot \vec{x}} \right] \]

\[ C_{z\bar{z}} = 2 \lim_{r \to \infty} \frac{1}{r} \partial_\mu x^\mu \partial_\nu x^\nu \sum_{\alpha = \pm} \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_q} \left[ \epsilon^{\alpha*}(\vec{q})a_{\alpha}(\vec{q})e^{-i\omega_q u - i\omega_q r(1-\cos \theta)} + h.c. \right] \]

\[ C_{\bar{z}z} = -\frac{i}{4\pi^2} \hat{\epsilon}_{\bar{z}z}^+ \int_0^\infty d\omega_q \left[ a_-(\omega_q \hat{x})e^{-i\omega_q u} - a_+(\omega_q \hat{x})^\dagger e^{i\omega_q u} \right] \]
\[ \langle \text{out}|a_-(q)S|\text{in} \rangle = \left( S^{(0)}- + S^{(1)}- \right) \langle \text{out}|S|\text{in} \rangle + O(\omega) \]

\[ S^{(0)}- = \sum_k \left( \frac{p_k \cdot \epsilon^-}{p_k \cdot q} \right)^2 \]

\[ S^{(1)}- = -i \sum_k \frac{p_k \mu \epsilon^{-\mu \nu} q^\lambda J_{k \lambda \nu}}{p_k \cdot q} \]
Soft Thm = Ward Id

\[ < \text{out}|Q^+[Y]S - SQ^-[Y]|\text{in}> = 0 \]

\[ Q^+[Y] = Q^+_S[Y] + Q^+_H[Y] \]

\[ Q^+_S[Y] = \frac{1}{2} \int_{I^+} du \, d^2z \, D^2Y \, u \partial_u C^z \]

\[ Q^+_H[Y] = \lim_{\Sigma \to I^+} \int_{\Sigma} d\Sigma \, \xi^\mu \eta^\nu \Sigma T_{\mu\nu}^M \]

\[ \langle \text{out}|a_{-}(q)S|\text{in}\rangle = \left( S^{(0)} - S^{(1)} \right) \langle \text{out}|S|\text{in}\rangle + O(\omega) \]

\[ S^{(1)} = -i \sum_k \frac{p_k \epsilon^{-\mu\nu} q^\lambda J_{k\lambda\nu}}{p_k \cdot q} \]
subleading soft graviton theorem \[=\] Ward identity for 4D superrotations \[=\] Ward identity for 2D stress tensor

the asymptotic symmetry is physical
subleading soft graviton theorem = Ward identity for 4D superrotations = Ward identity for 2D stress tensor

we should look for a 2D dual CFT
$4D \text{ Soft Mode} = 2D \text{ Current}$
The Celestial Conjecture:

scattering in asymptotically flat spacetimes is dual to a CFT living on the celestial sphere
4D Amplitude = 2D Correlator

4D Lorentz invariance = 2D global conformal symmetry

\[ \langle \mathcal{O}_{\Delta_1}^{\pm}(z_1, \bar{z}_1) \ldots \mathcal{O}_{\Delta_n}^{\pm}(z_n, \bar{z}_n) \rangle = \prod_{i=1}^{n} \int_{0}^{\infty} d\omega_i \omega_i^{\Delta_i - 1} \langle \text{out} | S | \text{in} \rangle \]

If we go to a boost basis, amplitudes transform as CFT correlators under the Lorentz group.
\[ m \neq 0 \]

\[ m = 0 \]

massive scalar

\[
\tilde{A}(\Delta_i, \bar{w}_i) \equiv \prod_{k=1}^{n} \int_{H_{d+1}} [d\hat{p}_k] G_{\Delta_k}(\hat{p}_k; \bar{w}_k) \ A(\pm m_i \hat{p}_i^\mu)
\]

\[
m = 0 \quad \tilde{A}(\Delta_i, \bar{w}_i) \equiv \prod_{k=1}^{n} \int_{0}^{\infty} d\omega_k \omega_k^{\Delta-1} A(\pm \omega_k q_k^\mu)
\]
Lorentz covariance guaranteed by this choice of wavepackets, with u-direction captured by a continuous spectrum

\[ \Delta = 1 + i\lambda \]

Meanwhile translations shift the weight

\[ p^\mu = q^\mu e^{\partial \Delta} \iff \Delta \mapsto \Delta + 1 \]
4D Amplitude = 4D Correlator

\[ \langle \text{out} | S | \text{in} \rangle_{\text{boost}} = \prod \lim_{r \to \infty} \int_{-\infty}^{\infty} d\nu_i \nu_i^{-\Delta_i} \langle r \Phi(\nu_1, r, z_1, \bar{z}_1) \ldots r \Phi(\nu_n, r, z_n, \bar{z}_n) \rangle \]

\[ \nu = \{ u, v \} \]
Carrollian CFT$\_3$  to bndy = on shell  Carrollian CFT$\_3$  to bndy = on shell  Celestial CFT$\_2$
Operator Spectrum

Fock Space $\leftrightarrow$ 4D Hilbert Space $\leftrightarrow$ 2D States $\leftrightarrow$ 2D Operators

$$\mathcal{O}_\Delta(z, \bar{z}) \equiv \int_{-\infty}^{\infty} du \, u^{-\Delta} \lim_{r \to \infty} [r^{\delta} \Phi(u, r, z, \bar{z})]$$

$$: \mathcal{O}^{(\rho)} \mathcal{O} :_\Delta (z, \bar{z}) \equiv \int_0^\infty d\omega \, \omega^{\Delta - \rho - 1} \int_0^\omega d\omega_1 \, \omega_1^{\rho - 1} a^\dagger(\omega_1, z, \bar{z}) a^\dagger(\omega - \omega_1, z, \bar{z})$$
\[ \Theta_{\Delta_1, +2}(z_1, \bar{z}_1) \Theta_{\Delta_2, +2}(z_2, \bar{z}_2) \sim -\frac{\kappa \bar{z}_{12}}{2 z_{12}} B(\Delta_1 - 1, \Delta_2 - 1) \Theta_{\Delta_1 + \Delta_2, +2}(z_2, \bar{z}_2) + \ldots , \]

\[ \Theta_{\Delta_1, +2}(z_1, \bar{z}_1) \Theta_{\Delta_2, -2}(z_2, \bar{z}_2) \sim -\frac{\kappa \bar{z}_{12}}{2 z_{12}} B(\Delta_1 - 1, \Delta_2 + 3) \Theta_{\Delta_1 + \Delta_2, -2}(z_2, \bar{z}_2) \]

\[ - \frac{\kappa \bar{z}_{12}}{2 \bar{z}_{12}} B(\Delta_1 + 3, \Delta_2 - 1) \Theta_{\Delta_1 + \Delta_2, +2}(z_2, \bar{z}_2) + \ldots , \]
Celestial OPE

Celestial OPE vs Feynman Diagrams
Celestial OPE vs Feynman Diagrams

\[ \mathcal{O}_{\Delta_1}(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2}(z_2, \bar{z}_2) \mathcal{O}_{\Delta_3}(z_3, \bar{z}_3) \sim \left( \frac{1}{z_{13}z_{23}z_{13}z_{23}} c_1 + \frac{(z_{23}z_{23})^{\Delta_1-2}}{(z_{13}z_{13})^{\Delta_1}} c_2 \right) \mathcal{O}_{\Delta_1+\Delta_2+\Delta_3-4}(z_3, \bar{z}_3) \]

\[ \mathcal{O}_{\Delta_2}(z_2, \bar{z}_2) \mathcal{O}_{\Delta_3}(z_3, \bar{z}_3) \sim \int d\Delta \frac{C_{\Delta_2,\Delta_3}^{\Delta}}{(z_{23}z_{23})^{\frac{1}{2}(\Delta_2+\Delta_3-\Delta)}} \mathcal{O}_{\Delta}(z_3, \bar{z}_3) \]

\[ + \int d\Delta d\sigma \frac{C_{\Delta_2,\Delta_3}^{(\Delta,\sigma)}}{(z_{23}z_{23})^{\frac{1}{2}(\Delta_2+\Delta_3-\Delta)}} \mathcal{R}_{\Delta}^{\sigma}(z_3, \bar{z}_3) \]
2D Radial Quantization → More Symmetries

For special weights, the SL(2,C) multiplets have primary descendants.

\[ H^k(z, \bar{z}) := \lim_{\epsilon \to 0} \epsilon \Theta_{k+\epsilon, 2}(z, \bar{z}), \quad \Delta = k = 2, 1, 0, -1, \ldots \]

Assuming these multiplets shorten, we have

\[ H^k(z, \bar{z}) = \sum_{m = \frac{k-2}{2}}^{\frac{2-k}{2}} \bar{z}^{-\frac{k-2}{2} - m} H_m^k(z), \quad w_n^p = \frac{1}{\kappa} (p - n - 1)! (p + n - 1)! H_n^{-2p+4} \]
2D Radial Quantization → More Symmetries

Complexifying the celestial sphere variables and defining a holomorphic commutator

\[ [A, B](z) = \frac{1}{2\pi i} \oint_{z} dw A(w)B(z) \]

gives a $L_{1+\infty}$ symmetry algebra for appropriately rescaled modes

\[ \left[ w^{p}_{n}, w^{q}_{m} \right](z) = \left[ n(q-1) - m(p-1) \right] w^{p+q-2}_{m+n}(z) \]
Do these symmetries beyond tree level, or the self-dual sector?

Can we realize them in the matter sector?

Can we really complexify the celestial sphere to define these currents?
WHY?
A Collision of Fields: Then

Our story starts with Strominger’s suggestion that...

...a series of separate studies from the sixties are secretly the same.

The relativists were systematizing what happens at long distances...

The quantum field theorists were worried about what was going on at low energies...

And, a little later, someone remembered there was a physical observable attached to each of these things....
A Collision of Fields: Then

- New Subleading Soft Graviton Thm.
- Superrotation Symmetry
- Spin Memory

Boost Basis

2D Stress Tensor

Asymptotic Symmetries

IR

Memory Effects

Soft Theorems
A Collision of Fields: Now

Soft Thm. = Ward Id.  IR Triangle  IR Div. as ASG Violation  OPE Bootstrap  $w_{1+\infty}$
2013  2015  2017  2019  2021  2023

Soft Operators as Currents  Soft Hair  Change of Basis  Amplitude Dictionary

two kinds of infinite dimensional symmetry enhancements
Holographic Symmetry Algebras

- Holomorphic collinear limits
- Self-dual gravity
- Twisted holography
- Conformally soft theorems
- $L_{w_{1+∞}}$ in gravity, YM analog, ...
- Gravitational phase space

A Collision of Fields: Now
A Collision of Fields: Now

- Holomorphic collinear limits
- Conformally soft theorems
- $Lw_{1+\infty}$ in gravity, YM analog, ...

**Holographic Symmetry Algebras**

- Amplitudes
- Covariant phase space
- Gravitational phase space

- Self-dual gravity
- Twisted holography

**Carrollian CFTs + conformal colliders**
A Collision of Fields: Soon

\[ \Lambda < 0 \]

\[ \Lambda = 0 \]

Conformal Bootstrap

It From Qubit

Corner

Amplitudes
A Collision of Fields: Soon

Celestial Symmetries

ANEC+

Quantum Info

Bootstrap
WHO?
use INSPIREHEP api... ask for author identifiers with hep-th primary paper in last 3 years with <10 authors ...

... merge with participant lists for major conferences (strings, string math, amplitudes, bootstrap, it from qubit) ...

Still an overestimate since this includes a lot of people entering the research pipeline...

... imposing an IR cutoff > 50 hep-th citations removes half

Let's look at these authors...

1. restrict to <10 author hep-th papers
2. Impose IR cutoff >50 citations
3. filter out anyone with <1/4 hep-th
4. extract the connected component

... and see who’s worked with who over the last 10 years.
can identify communities using Gephi

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<tr>
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Certainly. Here are the ten main themes distilled from the provided keywords:

1. Holography and AdS/CFT
2. Black Hole Physics
3. Conformal and Quantum Field Theories
4. String Theory and Dualities
5. Supersymmetry and Supergravity
6. Topological and Geometric Methods
7. Entanglement and Quantum Mechanics
8. Gauge Theories
9. Cosmology and the Universe
10. Quantum Gravity and General Relativity
can identify communities using Gephi

| 6 | (25.84%) |
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celestial people