Celestial Holography

from Bottom-up to Top-down

The Western Hemisphere Colloquium on Geometry and Physics

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Soft Thm=Ward Id

u-falloffs and
antipodal matching

Boost Basis

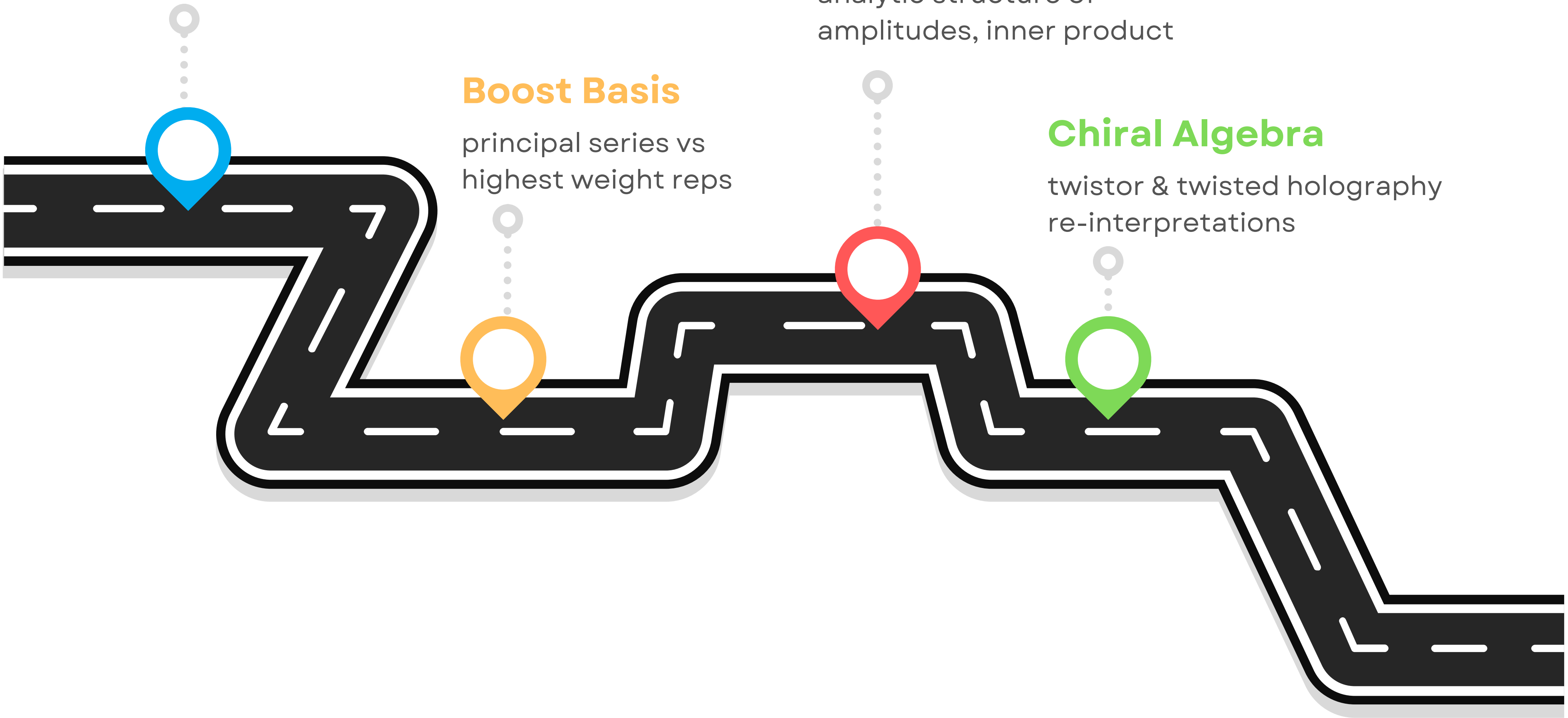
principal series vs
highest weight reps

Celestial OPE

analytic structure of
amplitudes, inner product

Chiral Algebra

twistor & twisted holography
re-interpretations



WHAT IS CELESTIAL HOLOGRAPHY?

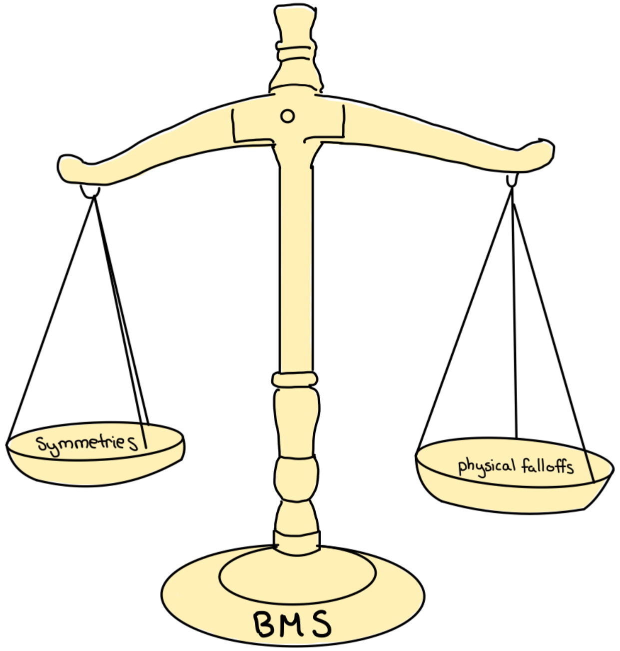
bh info +
string th

$\longrightarrow$

holographic
principle

$\Lambda = 0$ vs $\Lambda \to 0$

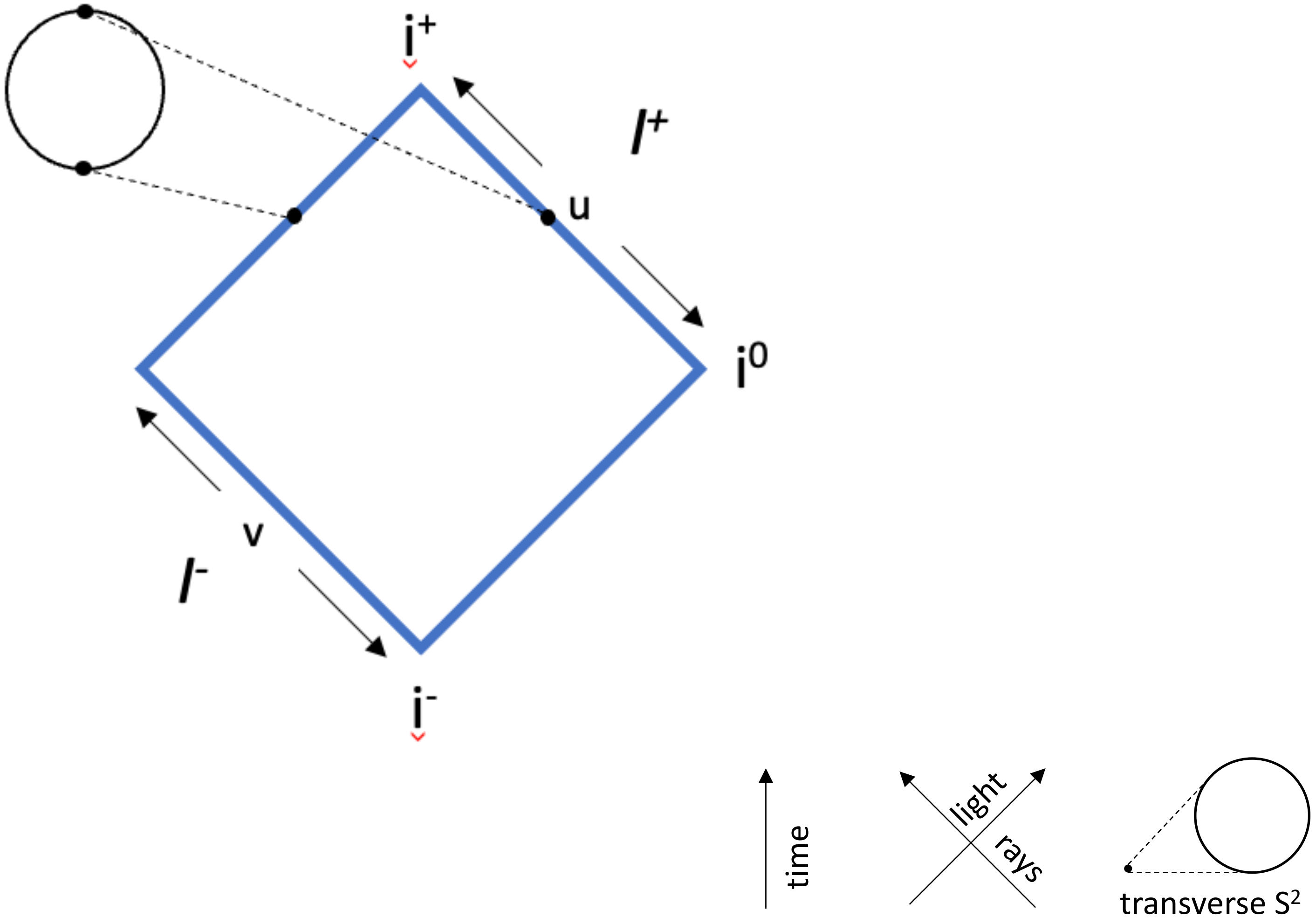
Lesson 1: BMS >> Poincare: $\Lambda = 0$ spacetimes have a much larger class of possible symmetries.



$$\text{ASG} = \frac{\text{allowed gauge symmetries}}{\text{trivial gauge symmetries}}$$

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$



$u = t - r$

$v = t + r$

In Bondi gauge the metric near future null infinity takes the form

$$\begin{aligned} ds^2 \quad &= -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} + 2\tfrac{m_B}{r}du^2 \\ &+\Big(rC_{zz}dz^2 + D^zC_{zz}dudz + \tfrac{1}{r}(\tfrac{4}{3}N_z - \tfrac{1}{4}\partial_z(C_{zz}C^{zz}))dudz + c.c.\Big) + \ldots \end{aligned}$$

Radiative Data

which is preserved by the residual diffeomorphisms

$$\begin{aligned} \xi^+ =&(1+\frac{u}{2r})Y^{+z}\partial_z - \frac{u}{2r}D^{\bar{z}}D_zY^{+z}\partial_{\bar{z}} - \frac{1}{2}(u+r)D_zY^{+z}\partial_r + \frac{u}{2}D_zY^{+z}\partial_u + c.c \\ &+ f^+\partial_u - \frac{1}{r}(D^zf^+\partial_z + D^{\bar{z}}f^+\partial_{\bar{z}}) + D^zD_zf^+\partial_r \end{aligned}$$

Superrotations

Supertranslations

Form this bulk analysis we land on the BMS group

$$
\begin{gathered}
[L_m, L_n] = (m-n)L_{m+n}, \quad [\bar{L}_m, \bar{L}_n] = (m-n)\bar{L}_{m+n}\,, \\
[L_n, P_{k,l}] = (\frac{1}{2}n - k)P_{k+n,l}, \quad [\bar{L}_n, P_{k,l}] = (\frac{1}{2}n - l)P_{k,l+n}\,, \\
[P_{m,n}, P_{k,l}] = 0.
\end{gathered}
$$

where $n, m \in \{-1,0,1\}$ & $k, l \in \left\{-\frac{1}{2}, \frac{1}{2}\right\}$ give the Poincare subalgebra

We can see this enhancement from the boundary perspective

$$ds^2 = -c^2 du^2 + dz d\bar{z}$$

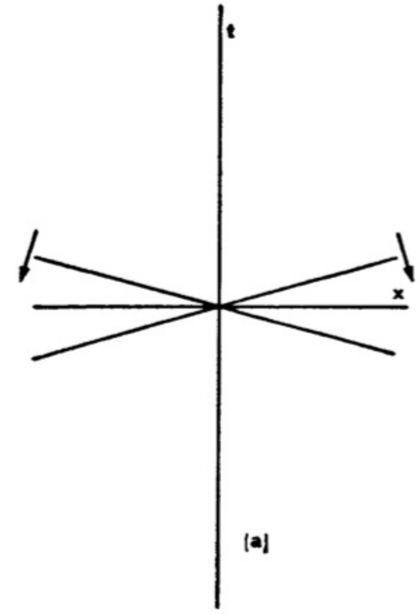
as a Carrollian limit of of a Lorentzian CFT_3.

$$\lim_{c \to 0} [\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = \alpha g_{\mu\nu}]$$

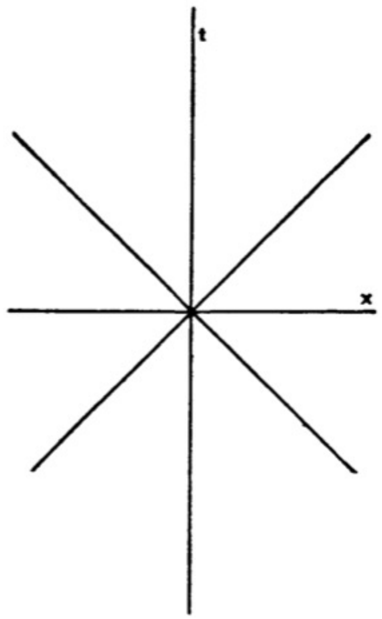
$$\lim_{c \to 0} c^2 [\nabla^\mu \xi^\nu + \nabla^\nu \xi^\mu = -\alpha g^{\mu\nu}]$$

$$ds^2 = -c^2 dt^2 + d\vec{x}^2$$

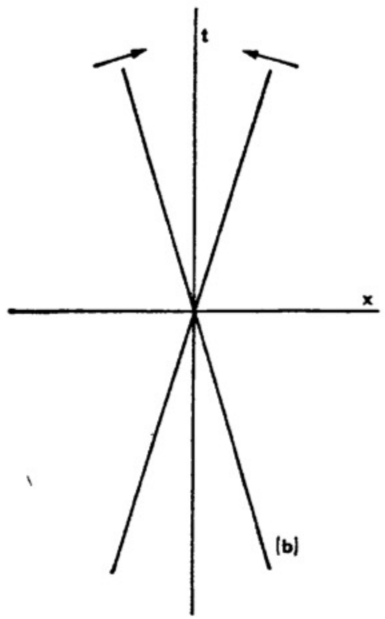
$c \to \infty$
Galilean



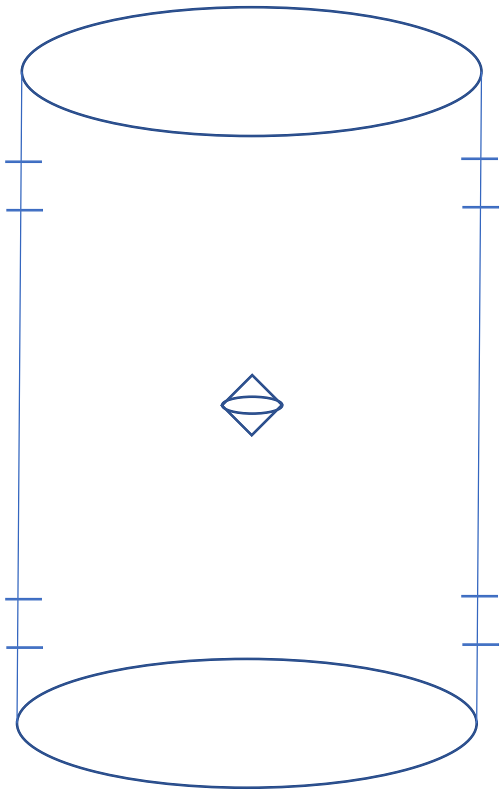
$c = 1$
Lorentzian



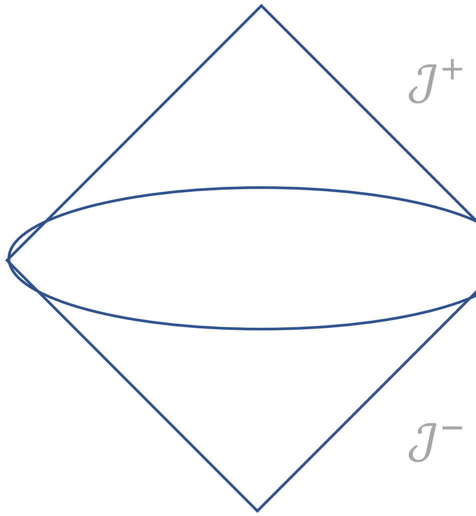
$c \to 0$
Carrollian

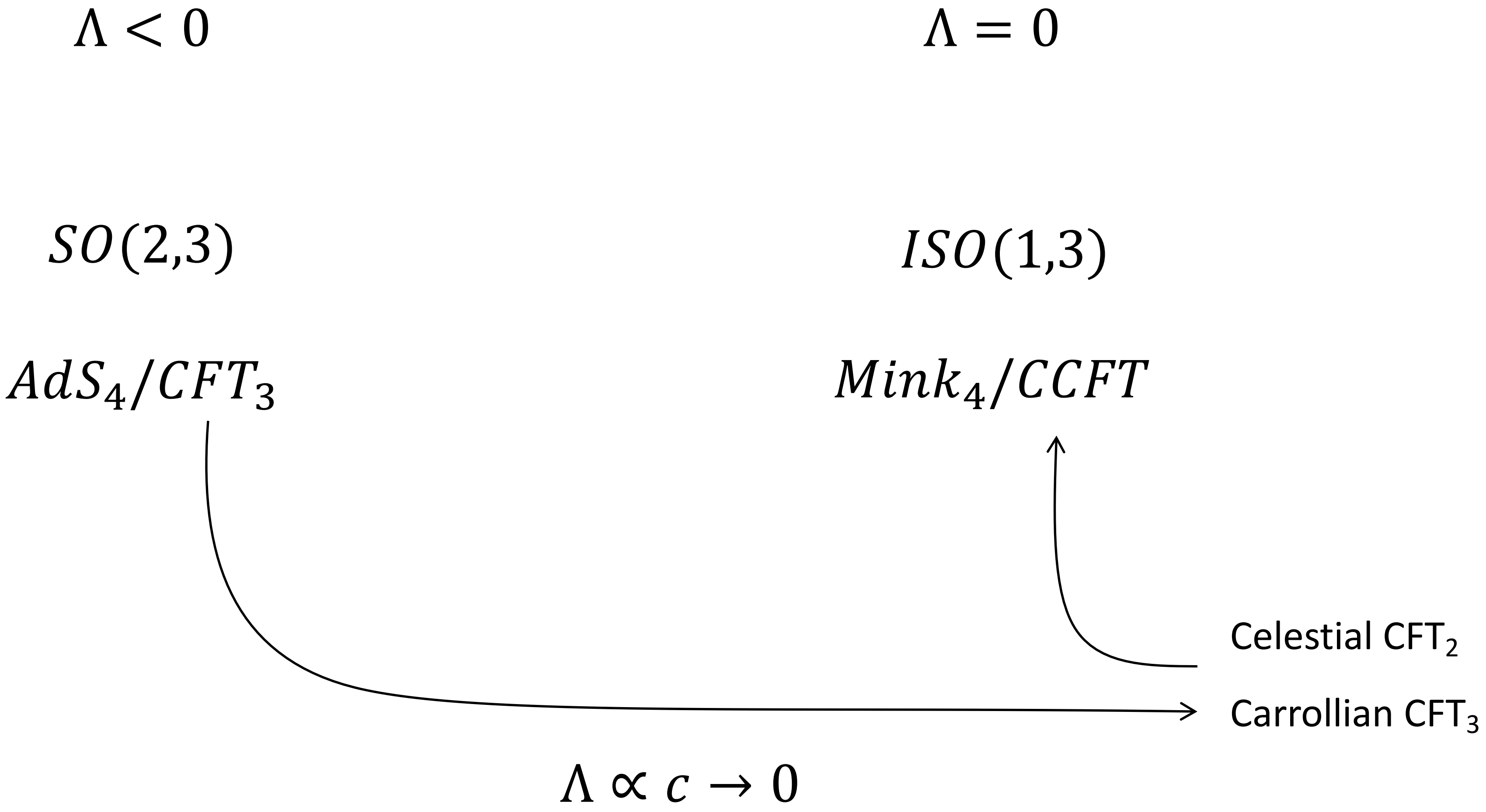


$\Lambda < 0$

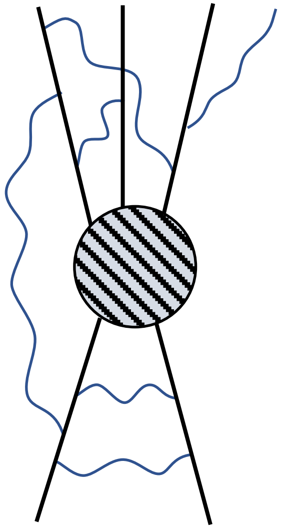


$\Lambda = 0$




$\Lambda < 0$
$\Lambda = 0$
$SO(2,3)$
$ISO(1,3)$
AdS_4/CFT_3
$Mink_4/CCFT$
Celestial CFT_2
Carrollian CFT_3
$\Lambda \propto c \to 0$

Lesson 2: These are indeed symmetries of the perturbative S-matrix!



$\Leftrightarrow$

Soft Thm = Ward Id

$$\langle out|Q^+[Y]\mathcal{S} - \mathcal{S}Q^-[Y]|in\rangle = 0$$

The free data for our solution takes the form

$$\{m_B(u_0, z, \bar{z}), N_z(u_0, z, \bar{z}), C_{zz}(u, z, \bar{z})\}$$

where the u dependence of the Bondi mass and angular momentum aspect are fixed by the constraint equations

$$n^\mu[G_{\mu\nu} - 8\pi G T_{\mu\nu}] = 0$$

The free data for our solution takes the form

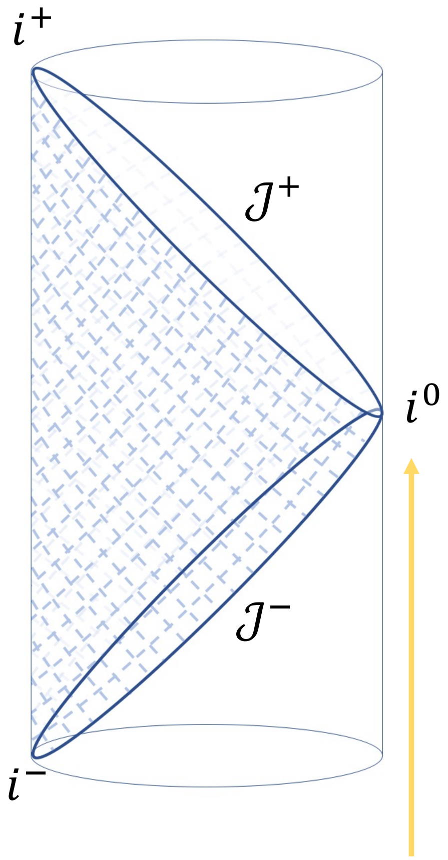
$$\{m_B(u_0, z, \bar{z}), N_z(u_0, z, \bar{z}), C_{zz}(u, z, \bar{z})\}$$

where the u dependence of the Bondi mass and angular momentum aspect are fixed by the constraint equations

$$\partial_u m_B = \frac{1}{4}[D_z^2 N^{zz} + D_{\bar{z}}^2 N^{\bar{z}\bar{z}}] - T_{uu}$$

1. $Q^+ = Q^-$ due to antipodal matching of m_B & N_z across i^0

2. Integration by parts turns the charges into fluxes



$$Q_f^+ = \frac{1}{8\pi G}\int_{\mathcal{I}^+_-} 2m_B f$$

$$8\pi G Q^+[Y] = \int_{\mathcal{I}^+} \sqrt{\gamma} d^2z du \left[-\frac{1}{2} D_z^3 Y^z u \partial_u C^{zz} + Y^z T_{uz} + u D_z Y^z T_{uu} + h.c. \right]$$

$$Q^+[Y] = Q_S^+[Y] + Q_H^+[Y]$$

$$C_{zz}|_{\mathcal{I}^+_-} = C_{zz}|_{\mathcal{I}^-_+}, \quad m_B|_{\mathcal{I}^+_-} = m_B|_{\mathcal{I}^-_+},$$

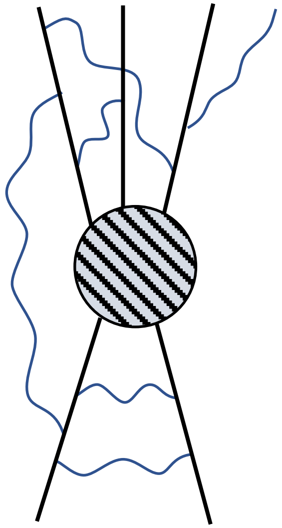
$$h_{\mu\nu}(x) = \sum_{\alpha=\pm} \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_q} \left[\epsilon^{\alpha *}_{\mu\nu}(\vec{q}) a_\alpha(\vec{q}) e^{iq\cdot x} + \epsilon^{\alpha}_{\mu\nu}(\vec{q}) a_\alpha(\vec{q})^\dagger e^{-iq\cdot x}\right]$$

radiative mode

$$C_{\bar{z}\bar{z}} = 2 \lim_{r\to\infty} \frac{1}{r} \partial_{\bar{z}} x^\mu \partial_{\bar{z}} x^\nu \sum_{\alpha=\pm} \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_q} \left[\epsilon^{\alpha *}_{\mu\nu}(\vec{q}) a_\alpha(\vec{q}) e^{-i\omega_q u - i\omega_q r(1-\cos\theta)} + h.c.\right]$$

saddle

$$C_{\bar{z}\bar{z}} = -\frac{i}{4\pi^2} \hat{\epsilon}^{+}_{\bar{z}\bar{z}} \int_0^\infty d\omega_q [a_-(\omega_q \hat{x}) e^{-i\omega_q u} - a_+(\omega_q \hat{x})^\dagger e^{i\omega_q u}]$$



$$\langle out|a_{-}(q)\mathcal{S}|in\rangle = \left(S^{(0)-} + S^{(1)-}\right)\langle out|\mathcal{S}|in\rangle + \mathcal{O}(\omega)$$

$$S^{(0)-} = \sum_k \frac{(p_k \cdot \epsilon^-)^2}{p_k \cdot q} \qquad S^{(1)-} = -i\sum_k \frac{p_{k\mu}\epsilon^{-\mu\nu}q^\lambda J_{k\lambda\nu}}{p_k \cdot q}$$

Soft Thm = Ward Id

$$\boxed{< out|Q^{+}[Y]\mathcal{S} - \mathcal{S}Q^{-}[Y]|in >= 0}$$

$$Q^{+}[Y] = Q^{+}_{S}[Y] + Q^{+}_{H}[Y]$$

$$Q^{+}_{S}[Y] = \frac{1}{2}\int_{\mathcal{I}^{+}} dud^{2}zD^{3}_{z}Y^{z}u\partial_{u}C^{z}{}_{\bar{z}}$$

$$Q^{+}_{H}[Y] = \lim_{\Sigma\to\mathcal{I}^{+}}\int_{\Sigma} d\Sigma\ \xi^{\mu}n^{\nu}_{\Sigma}T^{M}_{\mu\nu}$$

$$\langle out|a_{-}(q)\mathcal{S}|in\rangle = \left(S^{(0)-} + S^{(1)-}\right)\langle out|\mathcal{S}|in\rangle + \mathcal{O}(\omega)$$

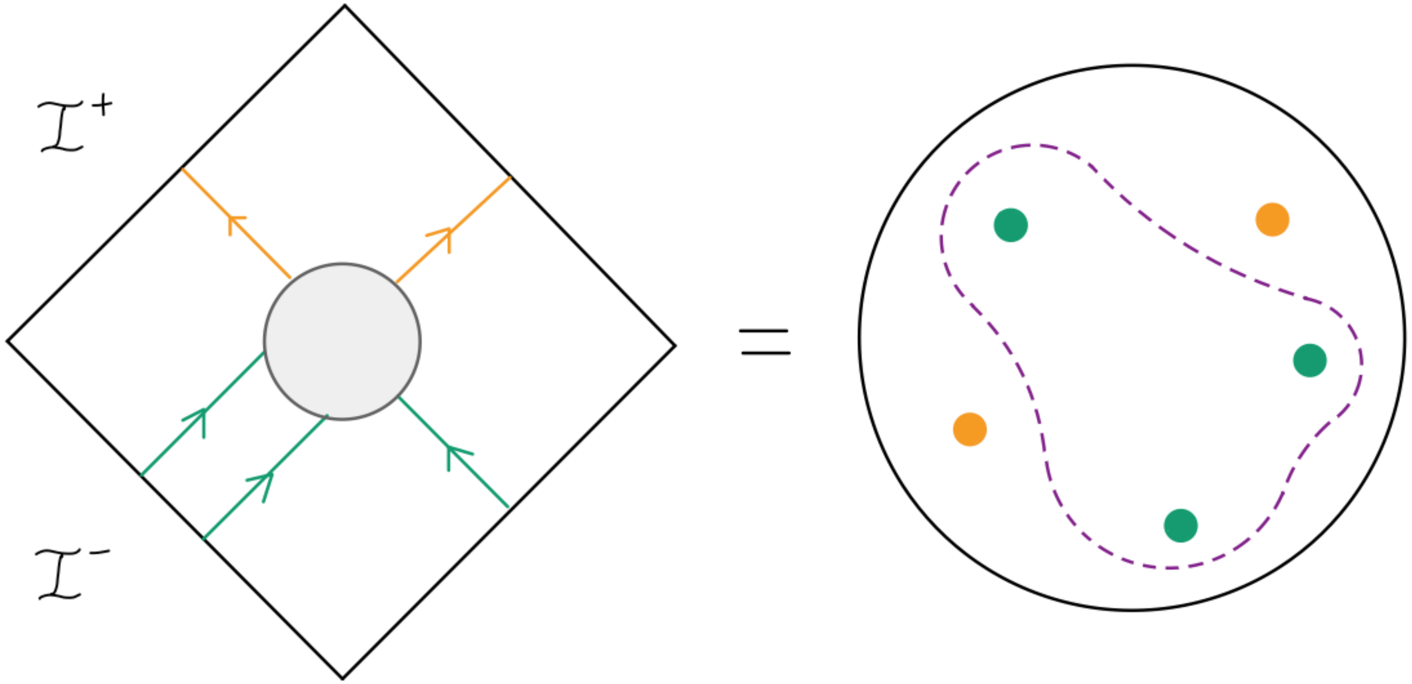
$$S^{(1)-} = -i\sum_{k}\frac{p_{k\mu}\epsilon^{-\mu\nu}q^{\lambda}J_{k\lambda\nu}}{p_{k}\cdot q}$$

subleading soft graviton theorem = Ward identity for 4D superrotations = Ward identity for 2D stress tensor

the asymptotic symmetry is physical

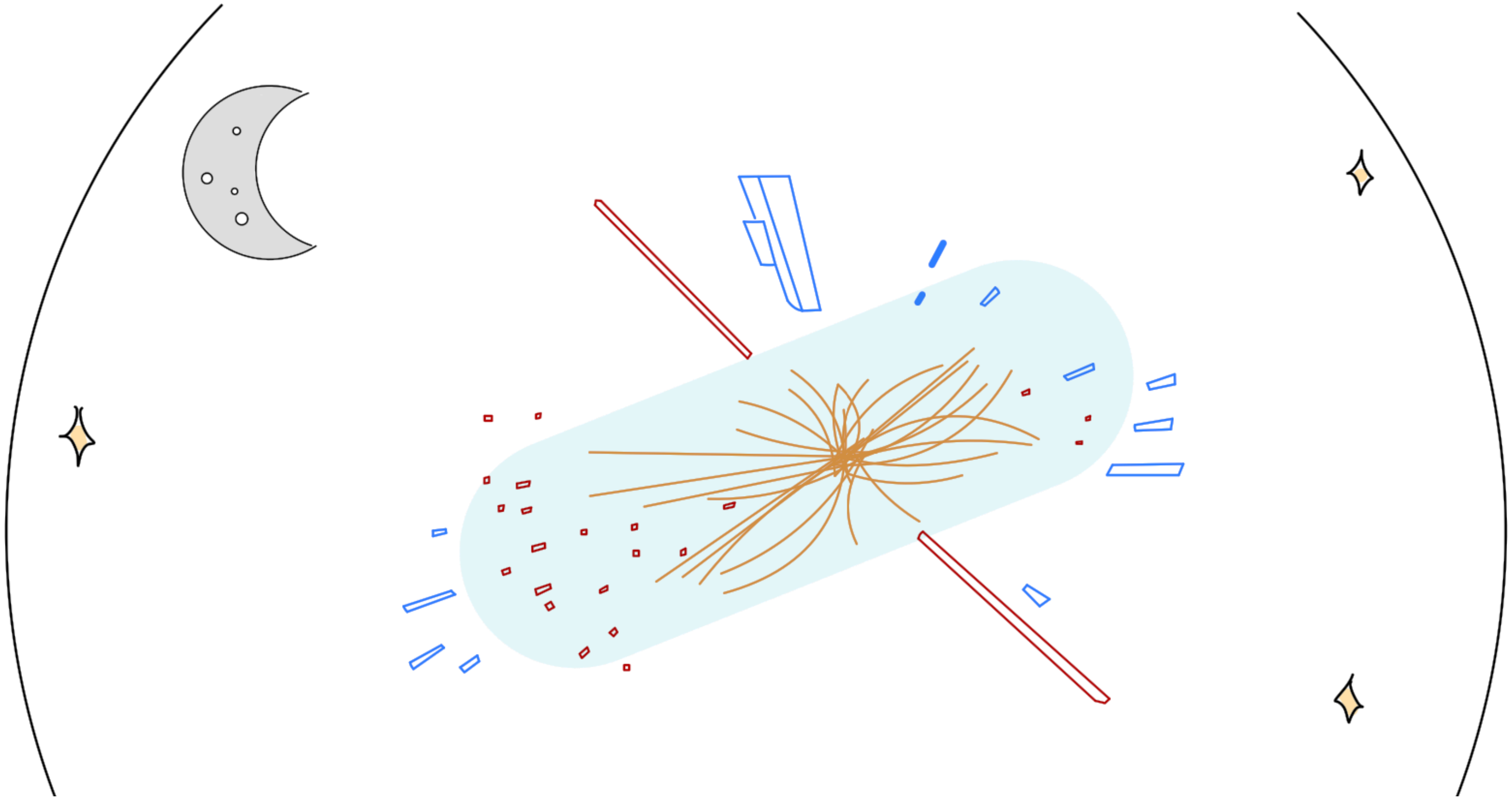
$$\text{subleading soft graviton theorem} \;=\; \underbrace{\text{Ward identity for 4D superrotations} \;=\; \text{Ward identity for 2D stress tensor}}_{\text{we should look for a 2D dual CFT}}$$

4D Soft Mode = 2D Current



The Celestial Conjecture:

scattering in asymptotically flat spacetimes is dual to a CFT living on the celestial sphere

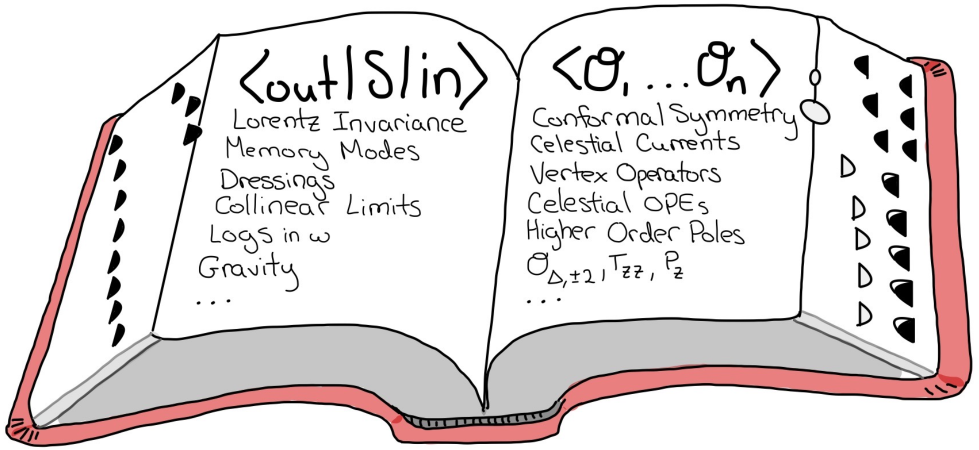


4D Amplitude = 2D Correlator

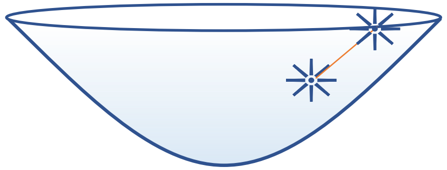
4D Lorentz invariance = 2D global conformal symmetry

$$\langle \mathcal{O}^{\pm}_{\Delta_1}(z_1, \bar{z}_1) \ldots \mathcal{O}^{\pm}_{\Delta_n}(z_n, \bar{z}_n) \rangle = \prod_{i=1}^{n} \int_0^\infty d\omega_i \omega_i^{\Delta_i - 1} \langle out | \mathcal{S} | in \rangle$$

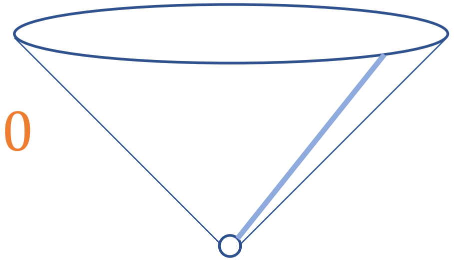
If we go to a boost basis, amplitudes transform as CFT correlators under the Lorentz group.



$m \neq 0$



$m = 0$



massive scalar

$$\widetilde{\mathcal{A}}(\Delta_i, \vec{w}_i) \equiv \prod_{k=1}^{n} \int_{H_{d+1}} [d\hat{p}_k]\, G_{\Delta_k}(\hat{p}_k; \vec{w}_k) \;\; \mathcal{A}(\pm m_i \hat{p}_i^{\mu})$$

$m = 0$

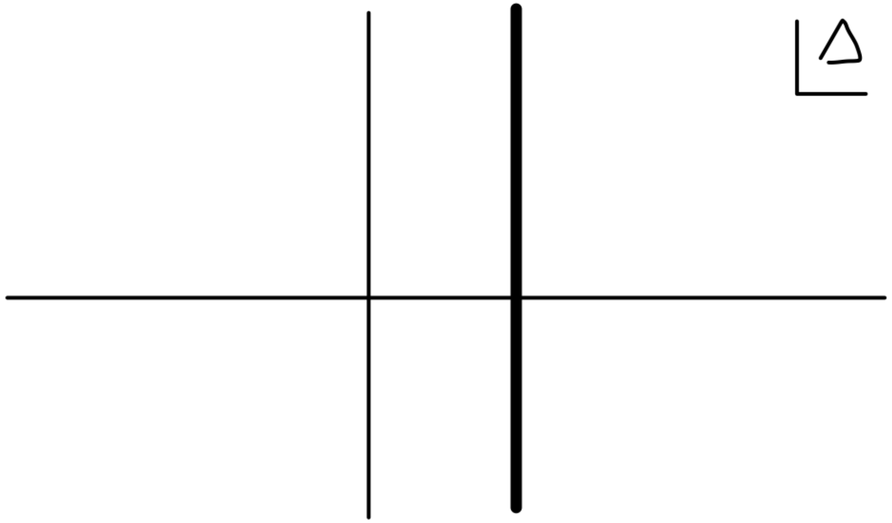
$$\widetilde{\mathcal{A}}(\Delta_i, \vec{w}_i) \equiv \prod_{k=1}^{n} \int_{0}^{\infty} d\omega_k \omega_k^{\Delta - 1}\, \mathcal{A}(\pm \omega_k q_k^{\mu})$$

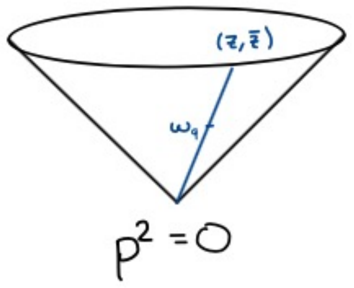
Lorentz covariance guaranteed by this choice of wavepackets, with u-direction captured by a continuous spectrum

$$\Delta = 1 + i\lambda$$

Meanwhile translations shift the weight

$$p^\mu = q^\mu e^{\partial_\Delta} \Leftrightarrow \Delta \mapsto \Delta + 1$$



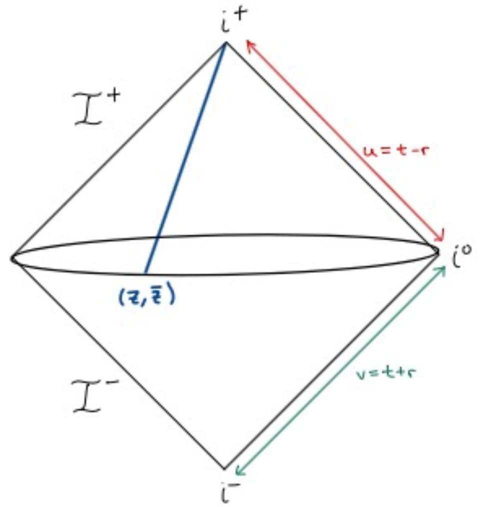


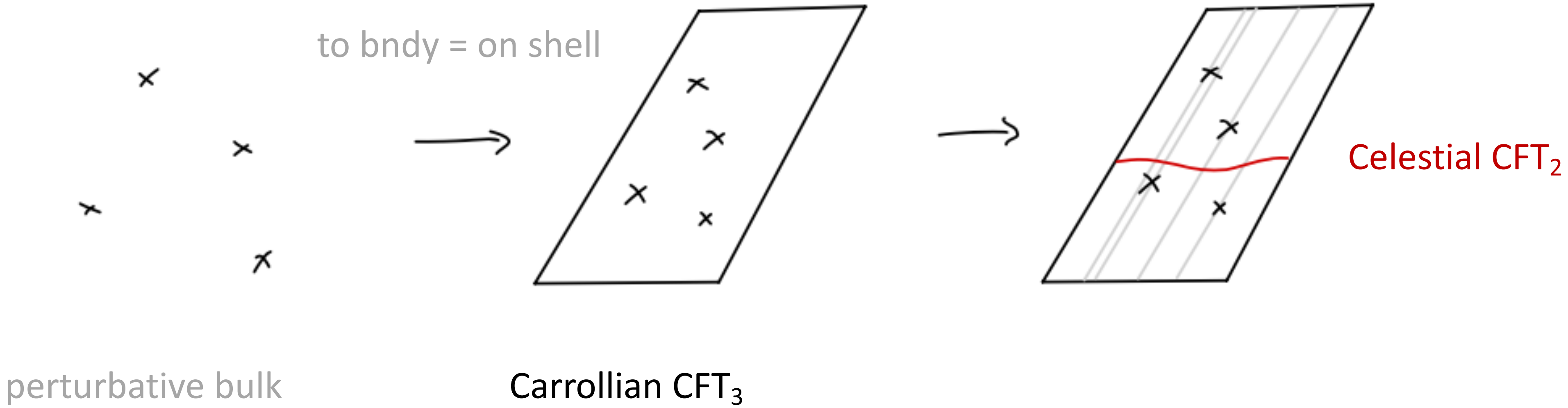
4D Amplitude = 4D Correlator

LSZ $\Leftrightarrow$ Extrapolate Dict.

$$\langle out|S|in\rangle_{boost} = \prod_i \lim_{r\to\infty} \int_{-\infty}^{\infty} \mathrm{d}\nu_i\, \nu_i^{-\Delta_i} \langle r\Phi(\nu_1, r, z_1, \bar{z}_1)...r\Phi(\nu_n, r, z_n, \bar{z}_n)\rangle$$

$$\nu = \{u, v\}$$




to bndy = on shell
perturbative bulk
Carrollian CFT_3
Celestial CFT_2

Operator Spectrum

Fock Space ↔ 4D Hilbert Space ↔ 2D States ↔ 2D Operators

$$\mathbb{O}_\Delta(z,\bar{z}) \equiv \int_{-\infty}^{\infty} \mathrm{d}u\, u^{-\Delta} \lim_{r\to\infty} \left[r^\delta \Phi(u,r,z,\bar{z})\right]$$

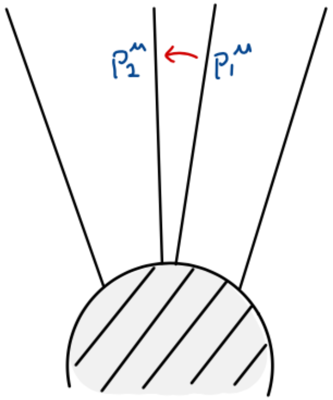
$$:\mathcal{O}^{(\rho)}\mathcal{O}:_\Delta(z,\bar{z}) \;\equiv\; \int_0^\infty d\omega\, \omega^{\Delta-\rho-1} \int_0^\omega d\omega_1\, \omega_1^{\rho-1}\, a^\dagger(\omega_1,z,\bar{z}) a^\dagger(\omega-\omega_1,z,\bar{z})$$

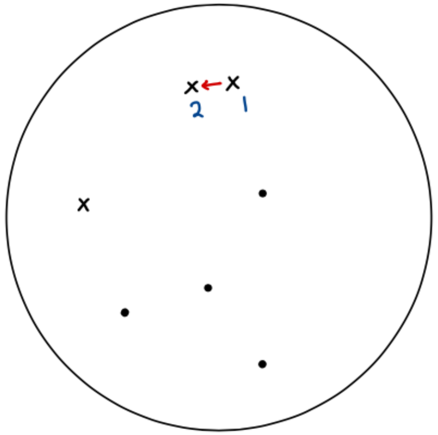
$\dots$

Collinear Limit = OPE

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,+2}(z_2,\bar{z}_2) \sim -\frac{\kappa}{2}\frac{\bar{z}_{12}}{z_{12}}B(\Delta_1-1,\Delta_2-1)\mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar{z}_2)+\ldots\,,$$

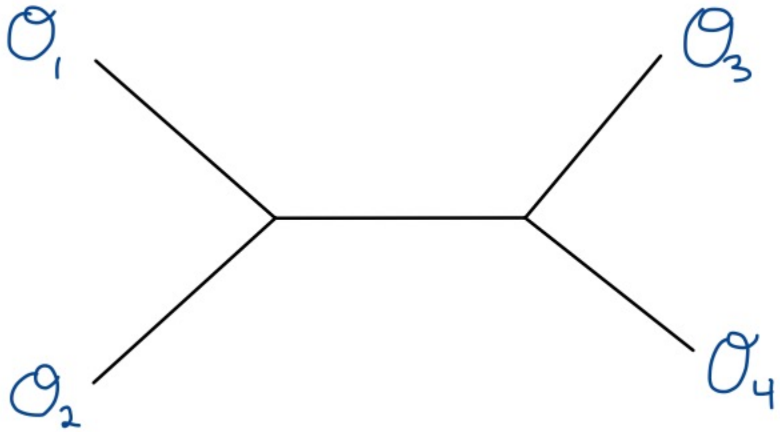
$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,-2}(z_2,\bar{z}_2) \sim -\frac{\kappa}{2}\frac{\bar{z}_{12}}{z_{12}}B(\Delta_1-1,\Delta_2+3)\mathcal{O}_{\Delta_1+\Delta_2,-2}(z_2,\bar{z}_2)$$
$$-\;\frac{\kappa}{2}\frac{z_{12}}{\bar{z}_{12}}B(\Delta_1+3,\Delta_2-1)\mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar{z}_2)+\ldots\,,$$

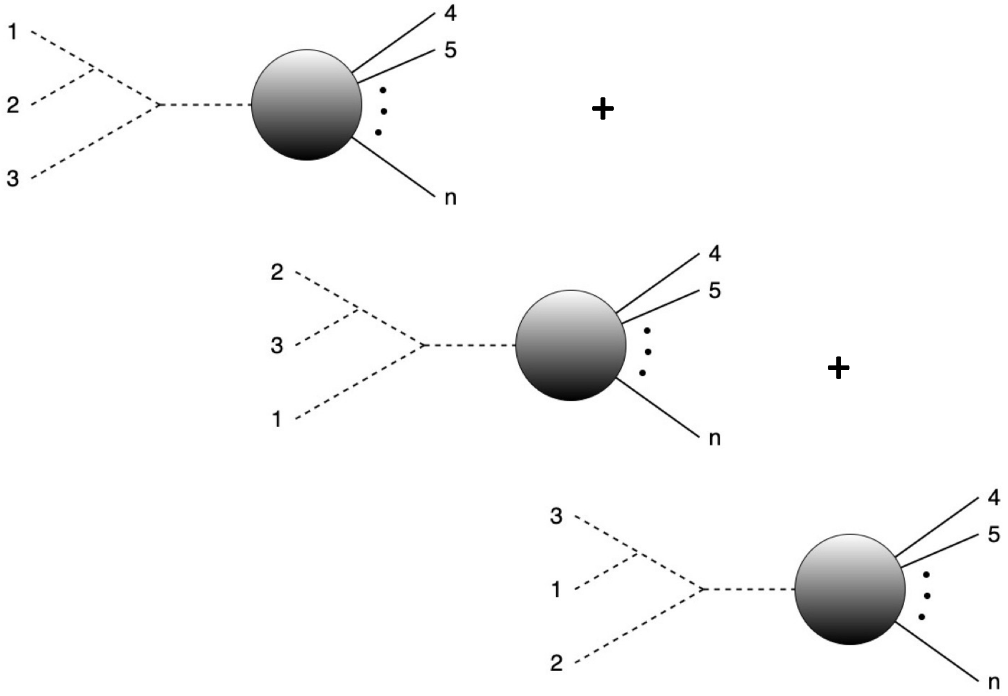




Celestial OPE

Celestial OPE vs Feynman Diagrams





Celestial OPE

Celestial OPE vs Feynman Diagrams

$$\mathcal{O}_{\Delta_1}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2}(z_2,\bar{z}_2)\mathcal{O}_{\Delta_3}(z_3,\bar{z}_3) \;\sim\; \left(\frac{1}{z_{13}z_{23}\bar{z}_{13}\bar{z}_{23}}\mathcal{C}_1 + \frac{(z_{23}\bar{z}_{23})^{\Delta_1-2}}{(z_{13}\bar{z}_{13})^{\Delta_1}}\mathcal{C}_2\right)\mathcal{O}_{\Delta_1+\Delta_2+\Delta_3-4}(z_3,\bar{z}_3)$$

$$\mathcal{O}_{\Delta_2}(z_2,\bar{z}_2)\mathcal{O}_{\Delta_3}(z_3,\bar{z}_3) \;\sim\; \int d\Delta\, \frac{{C_{\Delta_2,\Delta_3}}^{\Delta}}{(z_{23}\bar{z}_{23})^{\frac{1}{2}(\Delta_2+\Delta_3-\Delta)}}\,\mathcal{O}_{\Delta}(z_3,\bar{z}_3)$$

$$+\;\int d\Delta\, d\sigma\, \frac{{C_{\Delta_2,\Delta_3}}^{(\Delta,\sigma)}}{(z_{23}\bar{z}_{23})^{\frac{1}{2}(\Delta_2+\Delta_3-\Delta)}}\,\mathcal{R}^{\sigma}_{\Delta}(z_3,\bar{z}_3)$$

2D Radial Quantization → More Symmetries

For special weights, the SL(2,C) multiplets have primary descendants.

$$H^k(z,\bar{z}) \ := \ \lim_{\epsilon\to 0} \epsilon\, \mathcal{O}_{k+\epsilon,2}(z,\bar{z}), \quad \Delta = k \ = \ 2,1,0,-1,\dots$$

Assuming these multiplets shorten, we have

$$H^k(z,\bar{z}) \ = \ \sum_{m=\frac{k-2}{2}}^{\frac{2-k}{2}} \bar{z}^{-\frac{k-2}{2}-m}\, H^k_m(z)\ , \qquad\qquad w^p_n \ = \ \frac{1}{\kappa}\,(p-n-1)!(p+n-1)!\, H^{-2p+4}_n$$

2D Radial Quantization → More Symmetries

Complexifying the celestial sphere variables and defining a holomorphic commutator

$$[A, B](z) = \frac{1}{2\pi i} \oint_z dw\, A(w) B(z)$$

gives a $Lw_{1+\infty}$ symmetry algebra for appropriately rescaled modes

$$\left[w_n^p, w_m^q\right](z) = \left[n(q-1) - m(p-1)\right] w_{m+n}^{p+q-2}(z)$$

Do these symmetries beyond tree level, or the self-dual sector?

Can we realize them in the matter sector?

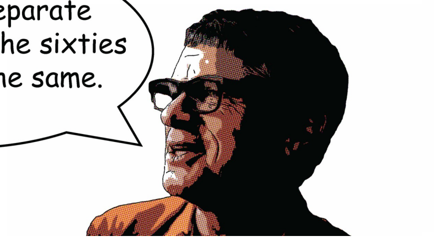
Can we really complexify the celestial sphere to define these currents?

WHY?

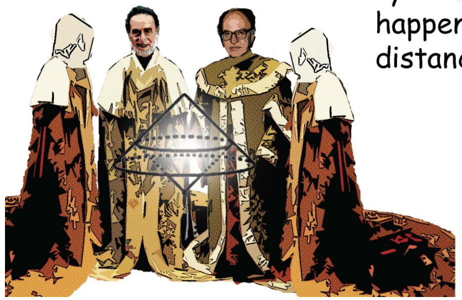
A Collision of Fields: Then

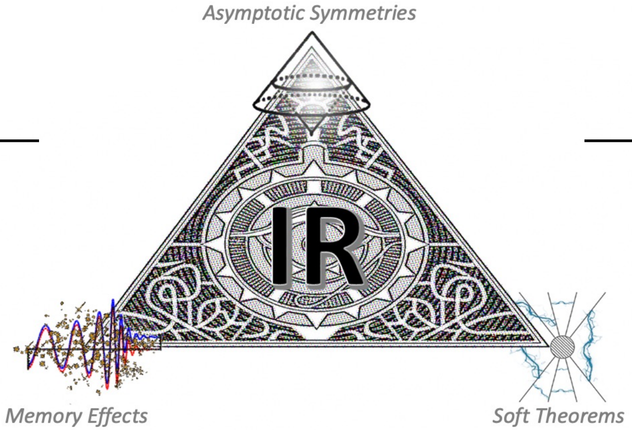
Our story starts with Strominger's suggestion that…

…a series of separate studies from the sixties are secretly the same.

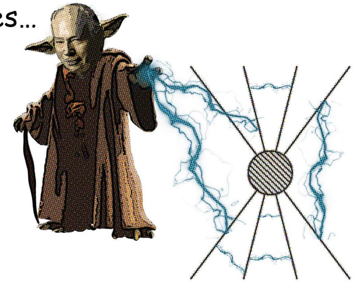


The relativists were systematizing what happens at long distances…





The quantum field theorists were worried about what was going on at low energies…



And, a little later, someone remembered there was a physical observable attached to each of these things….

A Collision of Fields: Then

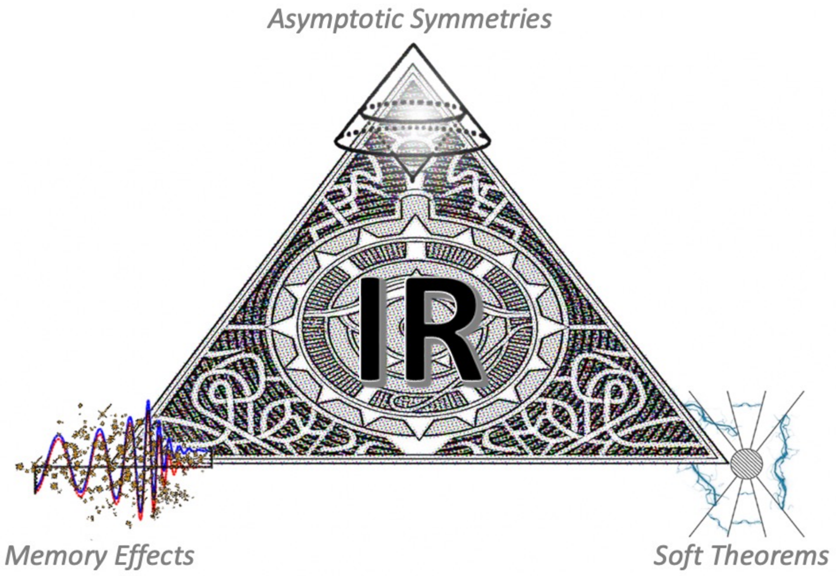
Boost Basis

2D Stress Tensor

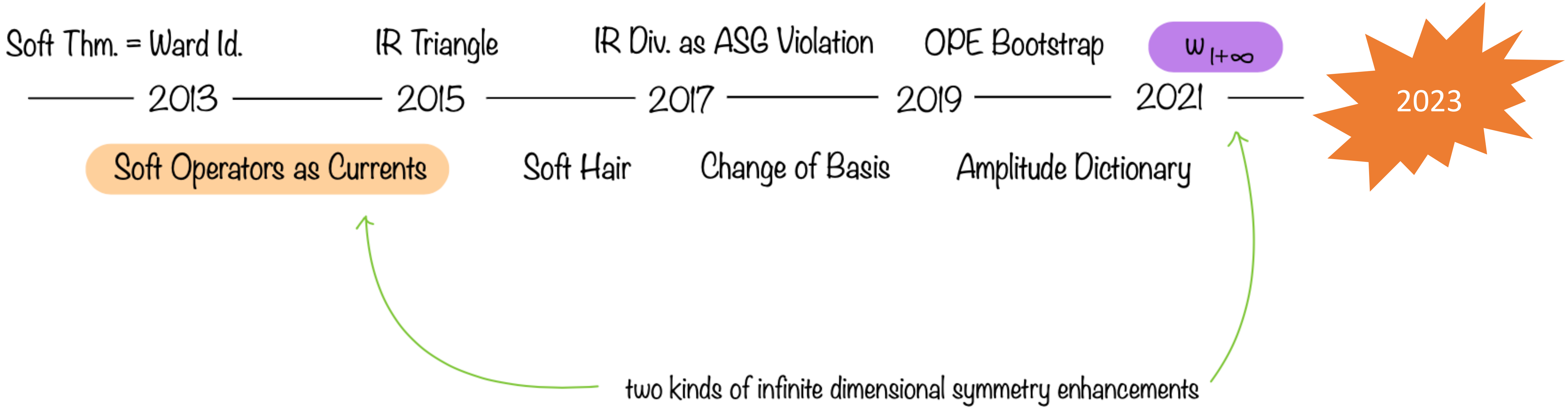
Superrotation
Symmetry

New Subleading
Soft Graviton Thm.

Spin Memory



A Collision of Fields: Now



A Collision of Fields: Now

conformally soft theorems

holomorphic collinear limits

$Lw_{1+\infty}$ in gravity, YM analog, …

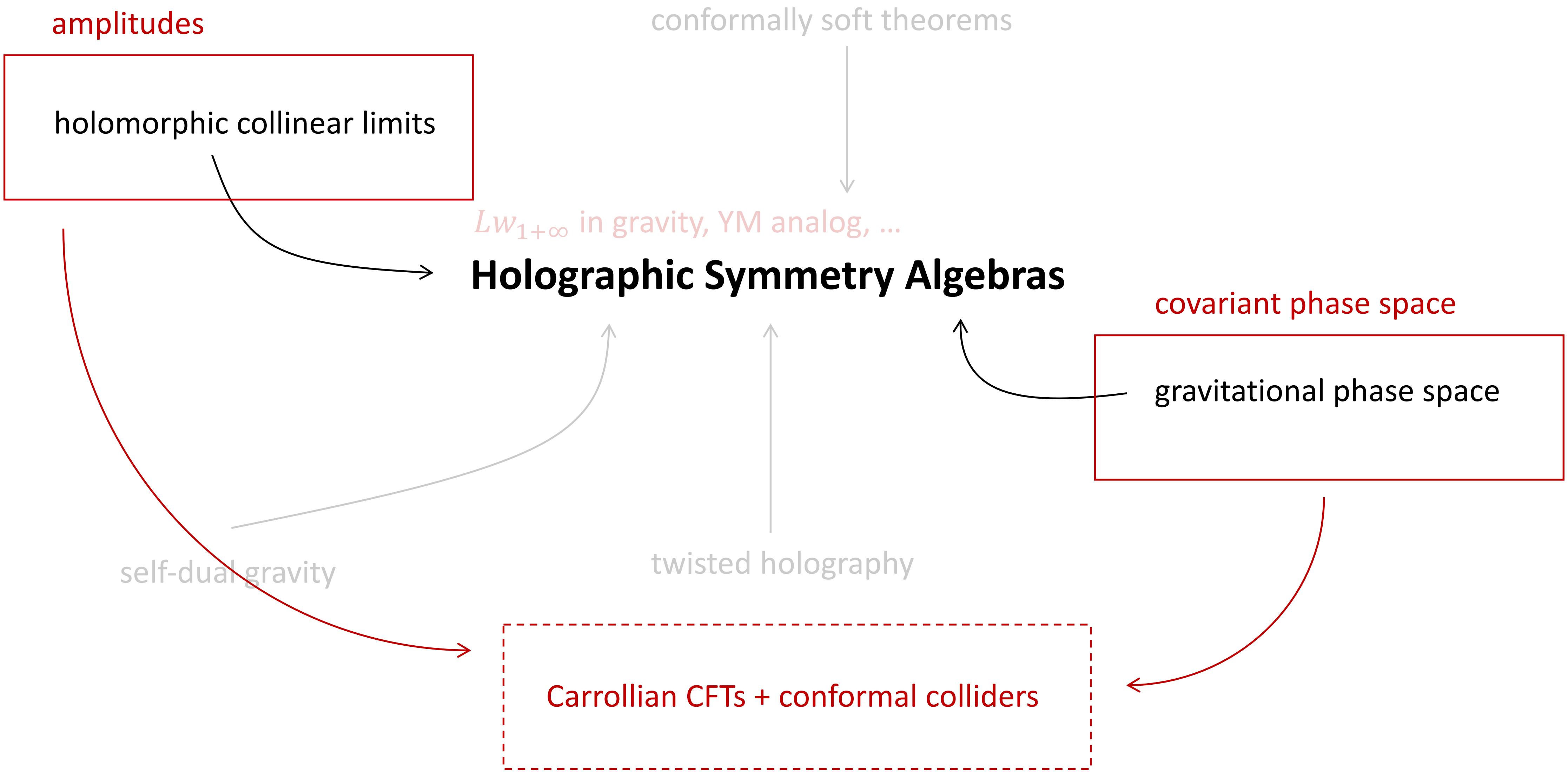
Holographic Symmetry Algebras

gravitational phase space

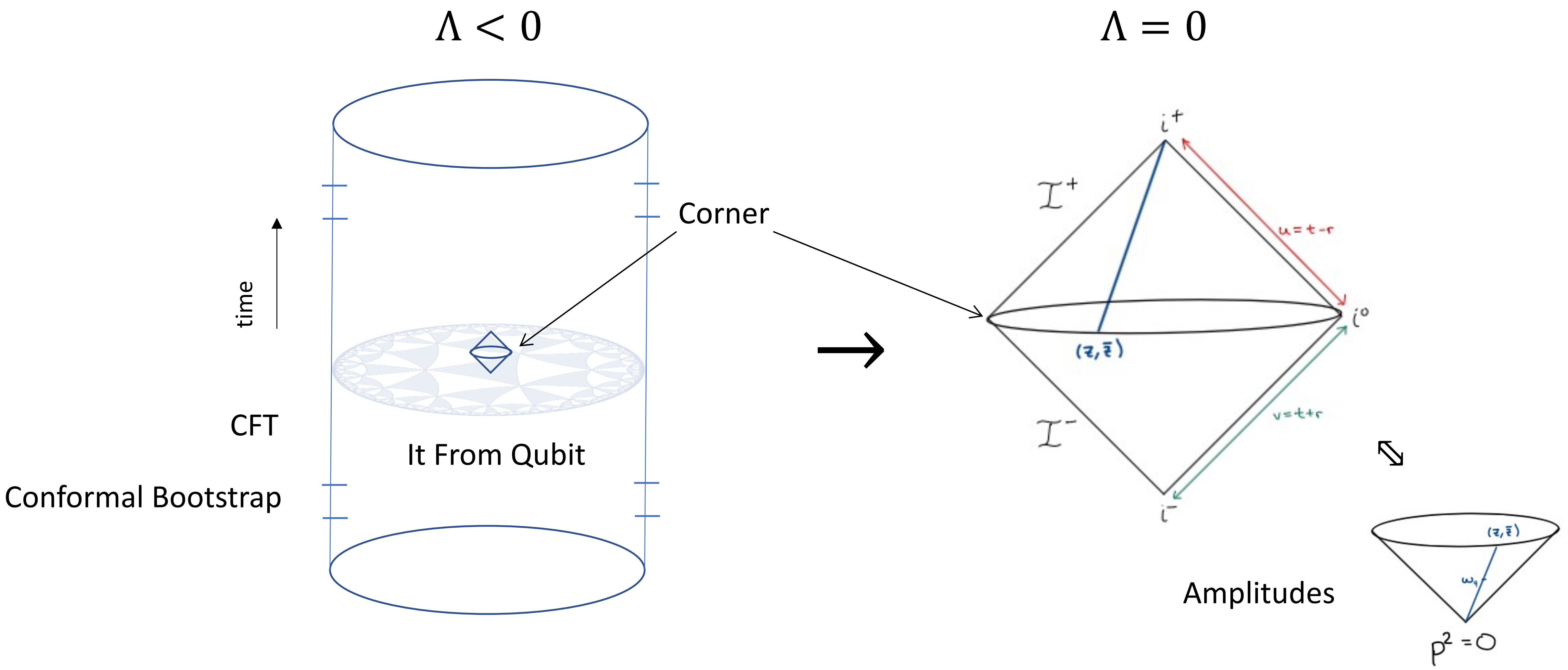
self-dual gravity

twisted holography

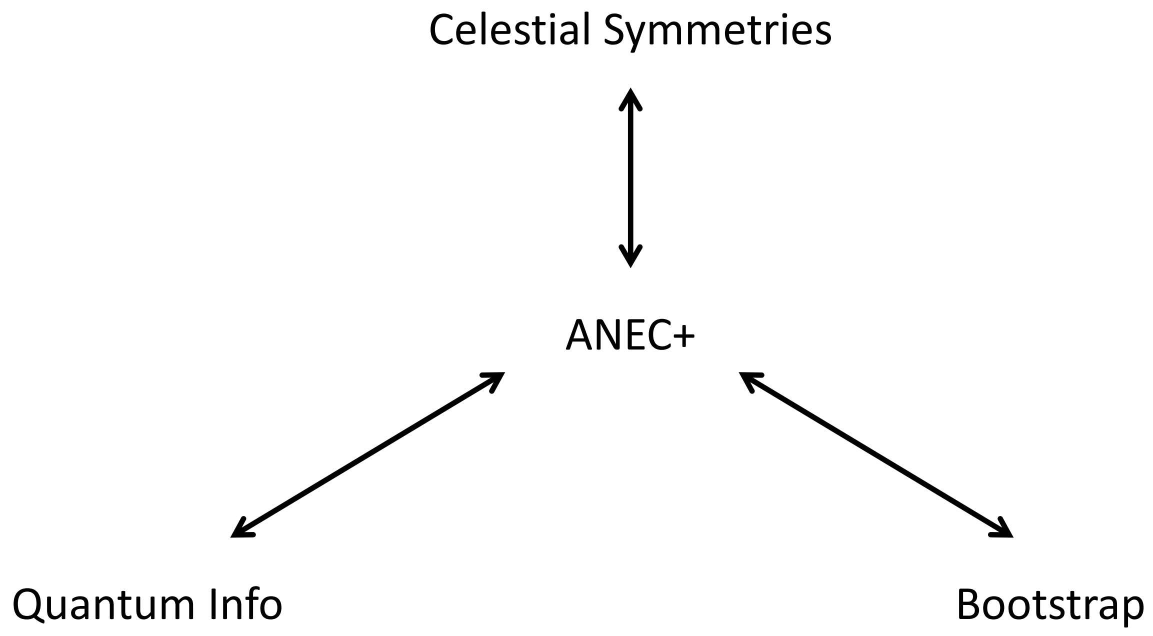
A Collision of Fields: Now



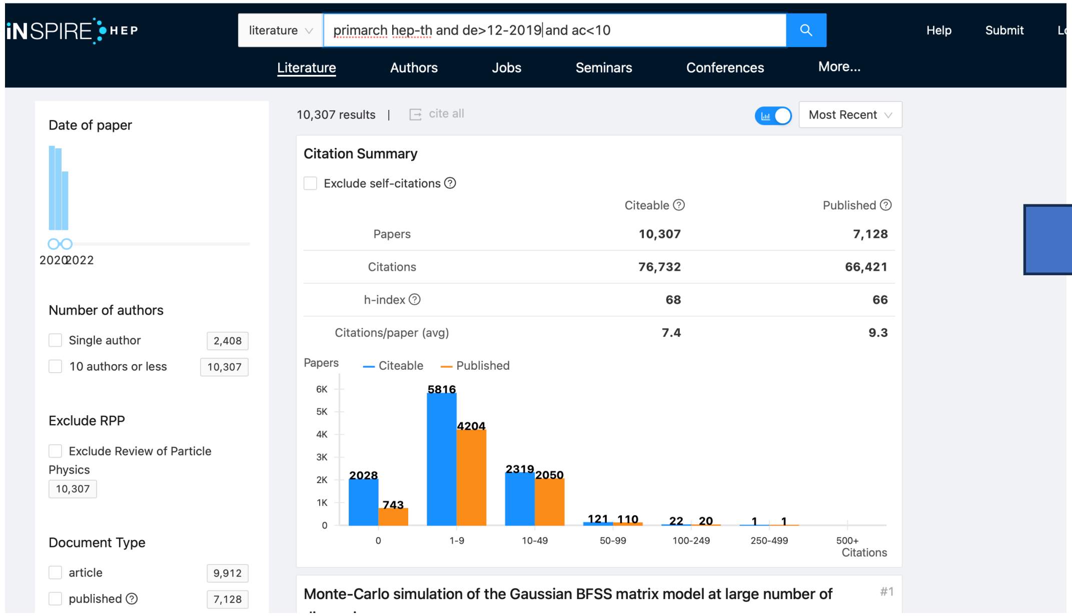
A Collision of Fields: Soon



A Collision of Fields: Soon

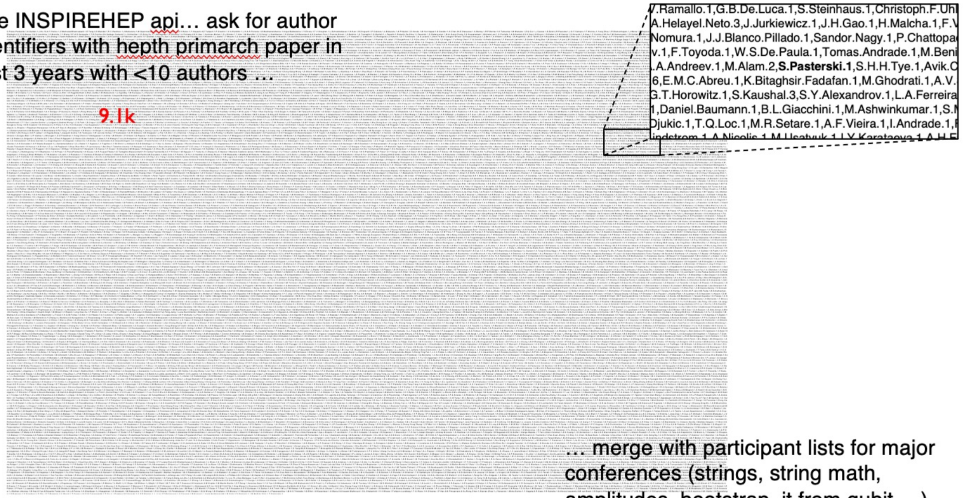


WHO?



use INSPIREHEP api… ask for author identifiers with hepth primarch paper in last 3 years with <10 authors …

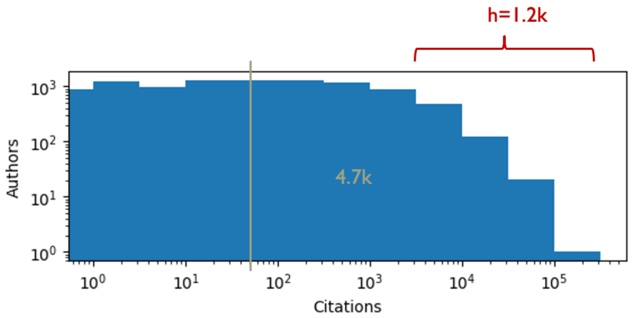
9.1k



… merge with participant lists for major conferences (strings, string math, amplitudes, bootstrap, it from qubit …)

9.5k

Still an overestimate since this includes a lot of people entering the research pipeline….



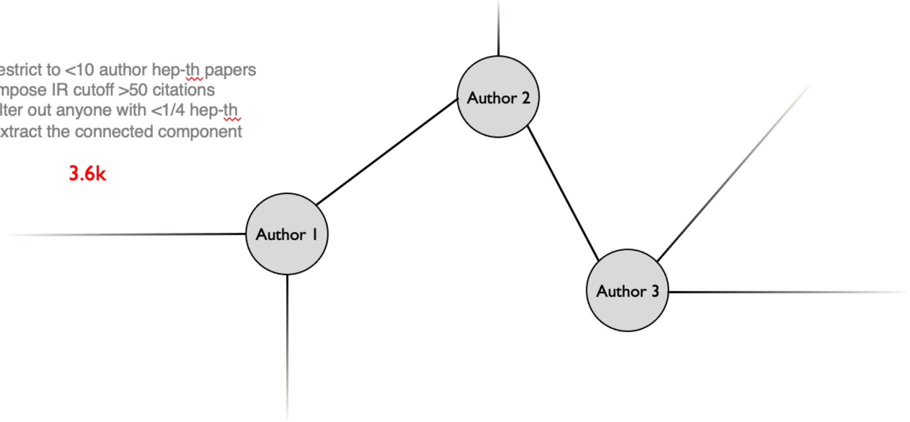
x authors with x or more citations

…. imposing an IR cutoff > 50 hep-th citations removes half

Let's look at these authors…

1. restrict to <10 author hep-th papers
2. Impose IR cutoff >50 citations
3. filter out anyone with <1/4 hep-th
4. extract the connected component

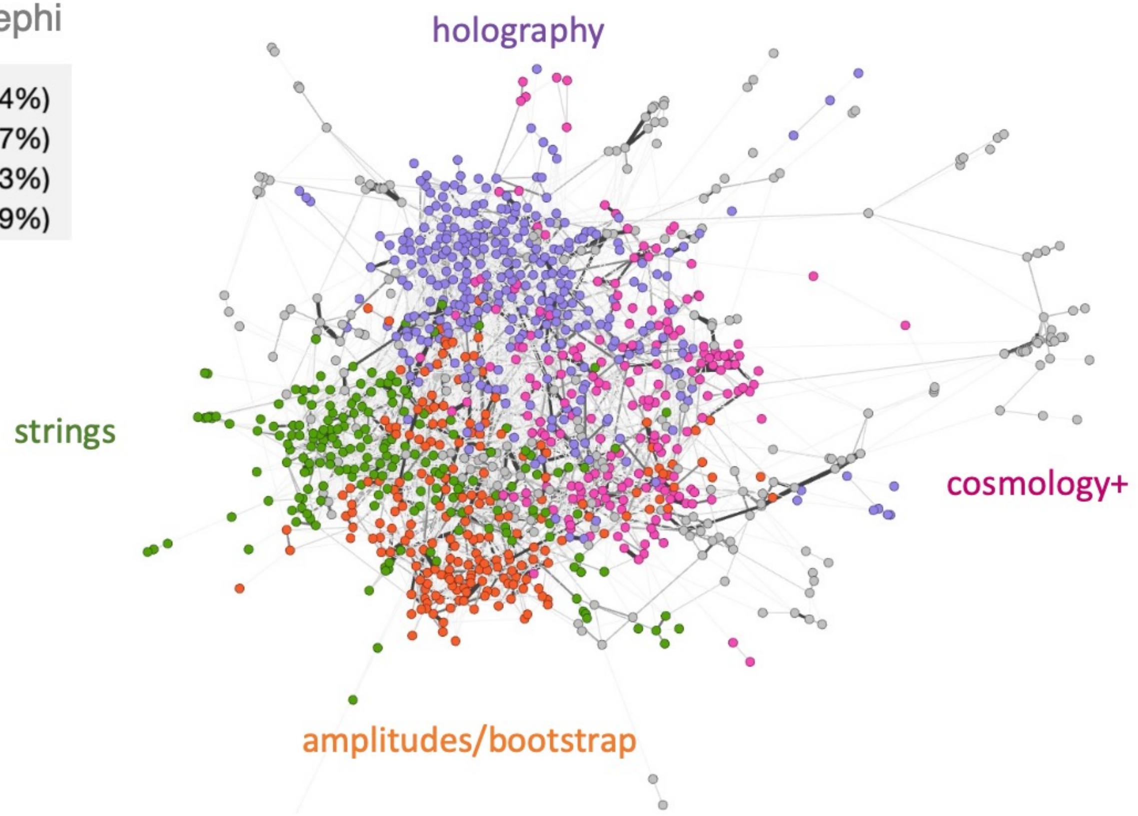
3.6k



… and see who's worked with who over the last 10 years.

can identify communities using Gephi

■	6	(25.84%)
■	1	(16.47%)
■	0	(16.13%)
■	2	(15.79%)



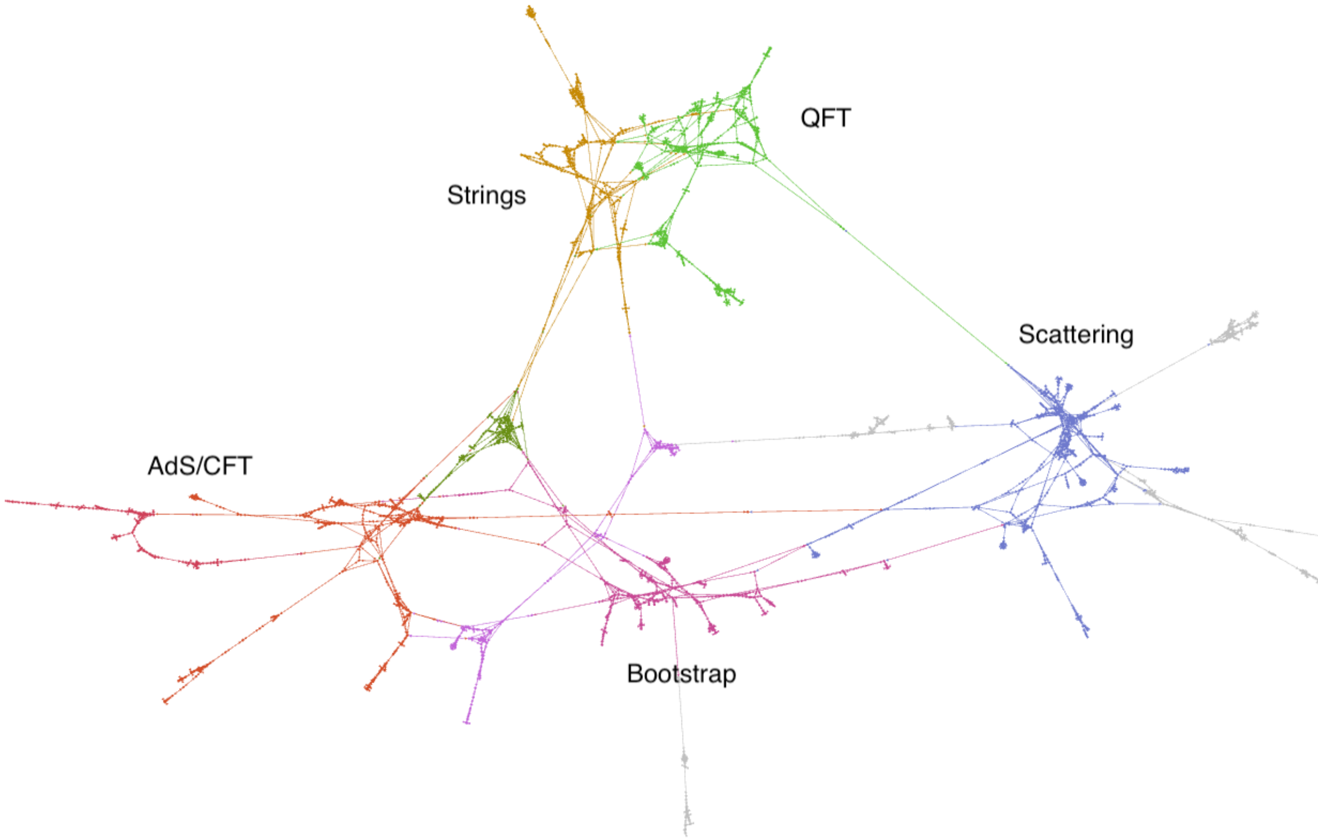
BAI	2022 Connected	Paper Clusters	Cluster Tags
E.Witten.1	1	[[2615434], [2154670, 2099463, 2029393, 2026876, 1997150, 1	['formalism for gravity', 'black hole threshold', 'joseph g polchinski', 'complex spacetime metrics', 'h
J.M.Maldacena.1	1	[[2616014, 2104727, 2104755], [2112370], [1923692, 1867137]]	['estimating global charge', 'accumulation point amplitudes', 'scale black holes']
C.Vafa.1	1	[[2616019, 2614218, 2613889, 2610564, 2154234, 2087695, 20	['dark dimension gravitons', 'string lamppost principle', 'bps black branes']
N.Seiberg.1	1	[[2514161], [2162567, 2154673, 2018548, 1897417, 1849552]]	['lieb schultz mattis', 'lifshitz theory tensor']
A.D.Linde.1	1	[[2593467, 2063978, 2055692, 2032108, 1949145, 1907935, 19	['hybrid cosmological attractors', 'goldstino condensation']
L.Susskind.1	1		
A.Strominger.1	1	[[2615487, 2070502, 1986080, 1973774, 1973157, 1866130, 18	['celestial conformal field', 'warped black holes', 'bfss matrix model', 'quantum encoder']
David.J.Gross.1	0		
N.Arkani.Hamed.1	1	[[2120761, 1990023], [2037717], [2020681], [1933887], [186695	['nonperturbative negative geometries', 'tropical geometry', 'tree level string', 'weak gravity conjectu
M.A.Shifman.1	0		
A.M.Polyakov.1	1		
S.Nojiri.1	0	[[2177393, 2139921, 2116125, 2083958, 2005610, 1994088, 19	['holographic dark energy', 'charged black holes', 'slow rotating black', 'horizon black holes', 'dark e
Sergio.Ferrara.1	0		
G.R.Dvali.1	0	[[2614902, 2172051, 1909144], [2176762, 2157990, 1991201, 1	['primordial black holes', 'black holes correspondence', 'efficient quantum computation']
I.R.Klebanov.1	1	[[2617473, 1869539], [2181809, 1975550], [2167953, 2094752,	['fermionic lattice models', 'critical field theories', 'discrete chiral symmetry']
A.A.Tseytlin.1	1	[[2617022], [2170581, 2122418, 2103461, 1866164, 1860825, 1	['derivative scalar theory', 'bps son loop', 'integrable supersymmetric deformations', 'defect qft',
A.I.Vainshtein.1	0	[[2058015], [1989007]]	['massless schwinger model', pes and hidden']
G.W.Gibbons.1	0		
G.Veneziano.1	1	[[2168963, 2063426, 2020643, 1856580, 1841069], [1869123]]	['scattering radiation reaction' gularity scattering maps']
Michael.Dine.1	0		
R.C.Myers.1	1	[[2166821, 2036329, 1960285], [2024912]]	['complexity equals', 'extrema nds made']
M.E.Shaposhnikov.1	0	[[2616328, 2054698, 2016467, 1904870, 1870526], [1955845, 1	['einstein cartan gravity', 'finite lan symanzik', 'reconciling resonant leptogenesis']
V.I.Zakharov.1	0	[[2178053, 2513318, 2162003, 2108965, 2028982, 2619683, 19	['gravitational chiral anomaly', amic critical exponent', 'quantized vortices', 'phase modulation r
L.J.Dixon.1	1	[[2610588, 2514138, 2120761, 2072456, 2072539, 2058002, 19	['super yang mills', 'point integ integrable']

GPT-4

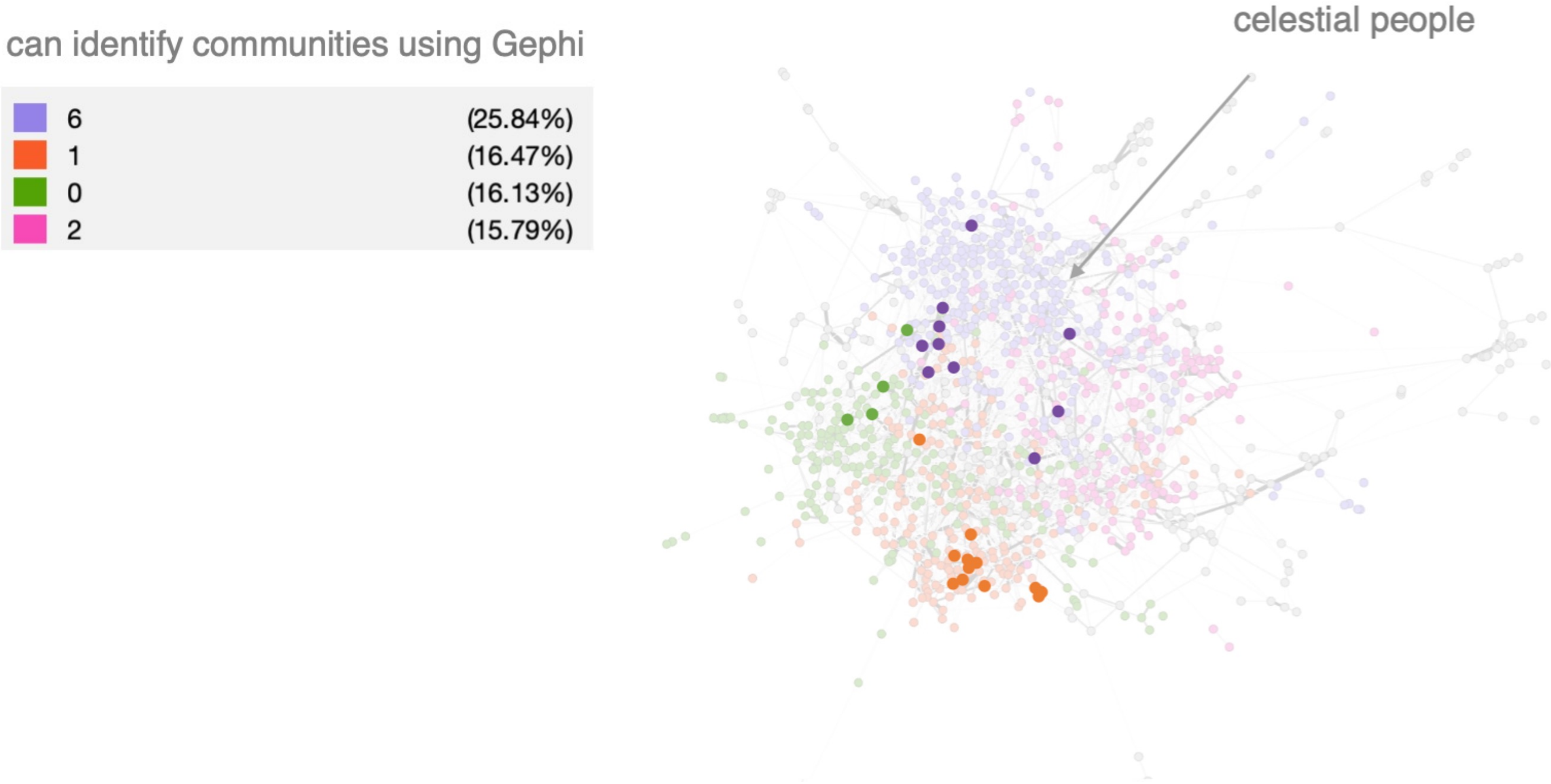
S below I will copy in some keywords from recent hep-th papers can you distill 10 main themes. feel free to use your knowledge of the keywords

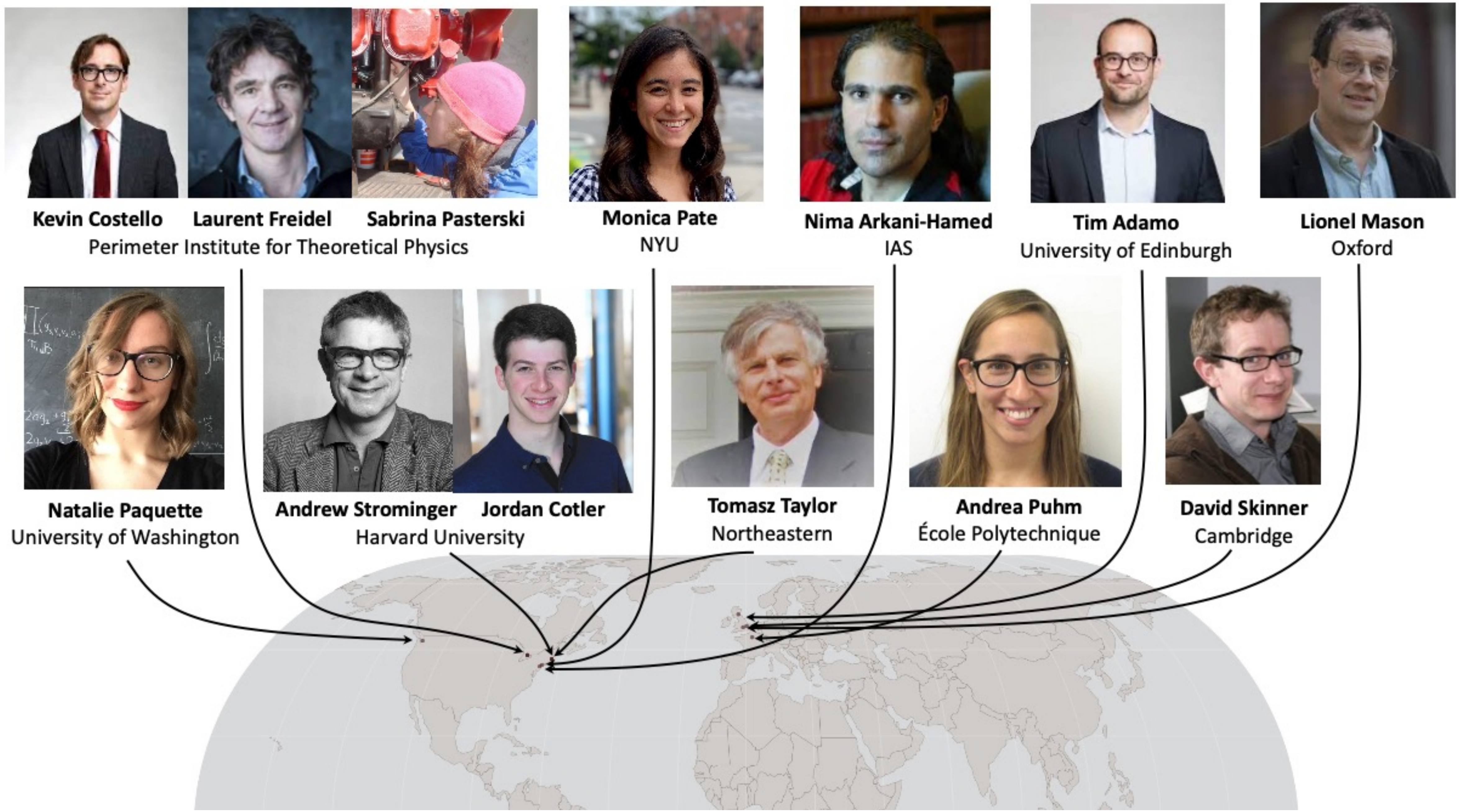
Certainly. Here are the ten main themes distilled from the provided keywords:

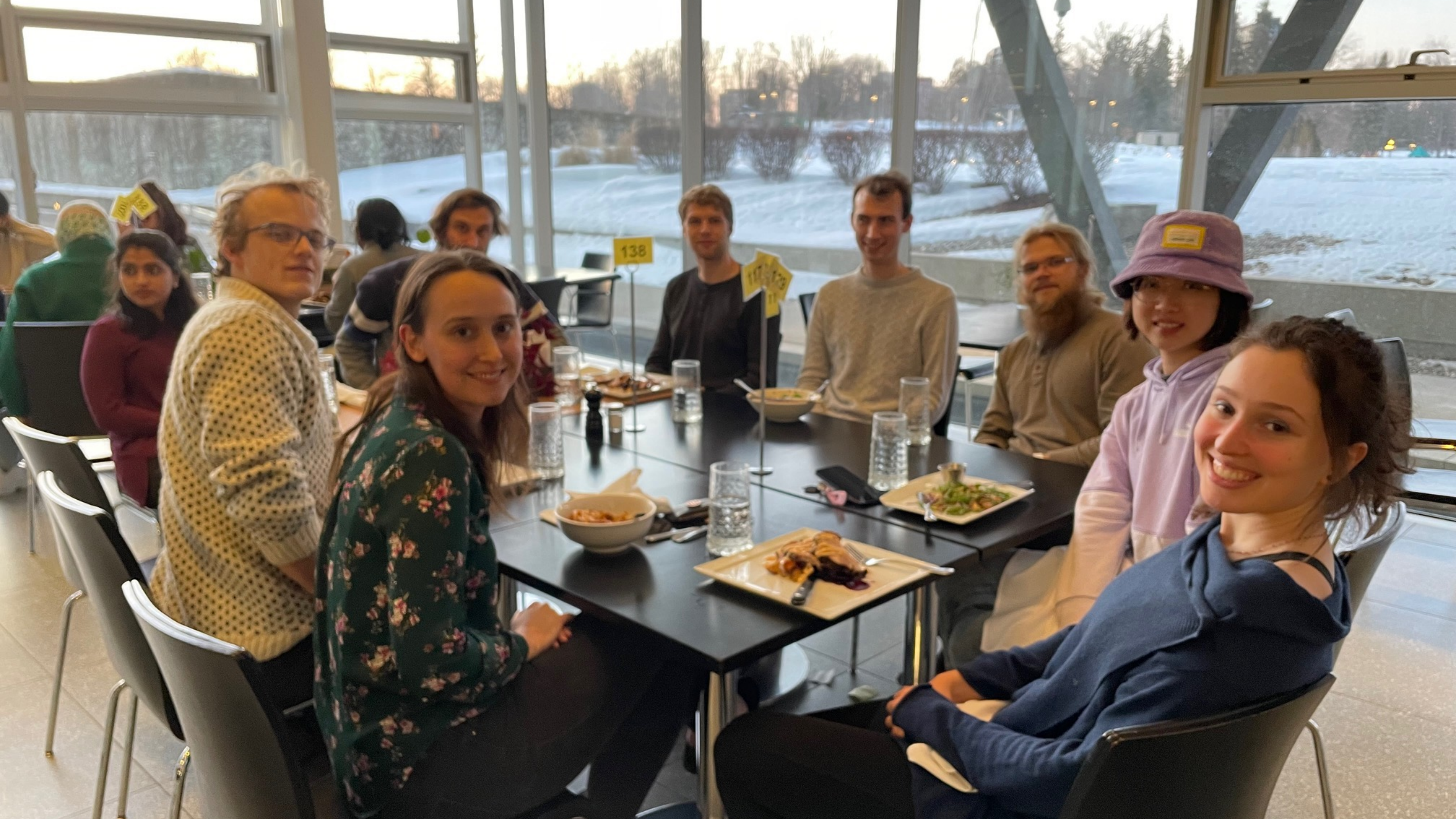
1. **Holography and AdS/CFT**
2. **Black Hole Physics**
3. **Conformal and Quantum Field Theories**
4. **String Theory and Dualities**
5. **Supersymmetry and Supergravity**
6. **Topological and Geometric Methods**
7. **Entanglement and Quantum Mechanics**
8. **Gauge Theories**
9. **Cosmology and the Universe**
10. **Quantum Gravity and General Relativity**



can identify communities using Gephi




Kevin Costello
Laurent Freidel
Sabrina Pasterski
Perimeter Institute for Theoretical Physics
Monica Pate
NYU
Nima Arkani-Hamed
IAS
Tim Adamo
University of Edinburgh
Lionel Mason
Oxford
Natalie Paquette
University of Washington
Andrew Strominger
Jordan Cotler
Harvard University
Tomasz Taylor
Northeastern
Andrea Puhm
École Polytechnique
David Skinner
Cambridge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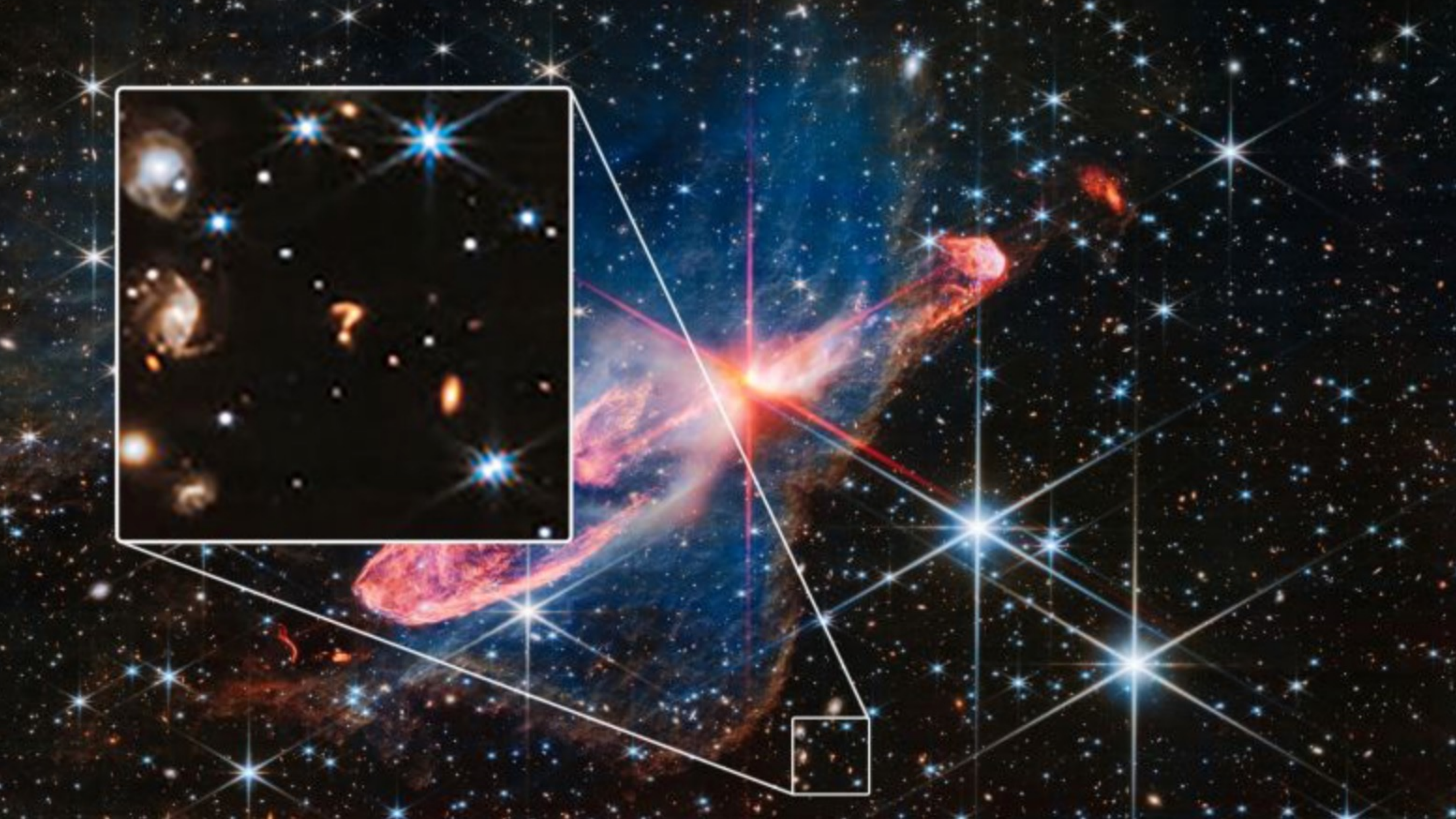


EFFECTIVE FIELD THEORY
ADS/CFT CORRESPONDENCE
COSMOLOGY
BLACK HOLES
MEMORY
STRING THEORY
GENERAL RELATIVITY
GRAVITATION