Middle intersection forms on singular elliptic Calabi-Yau fourfolds and applications to the standard model and mirror symmetry

Western Hemisphere Colloquium on Geometry and Physics

March 28, 2022

Washington (Wati) Taylor, MIT

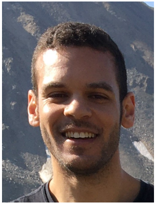
Based on recent and upcoming work with:

P. Jefferson, M. Kim, S.Y. Li, P. Oehlmann, A. Turner

Outline

1. Middle cohomology on elliptic Calabi-Yau fourfolds and chiral matter in 4D F-theory models
2. Standard model constructions from direct tuning and E_7 breaking
3. Mirror symmetry and elliptic CY fourfolds

1. Middle cohomology on elliptic Calabi-Yau fourfolds and chiral matter in 4D F-theory models



Patrick Jefferson

Andrew Turner

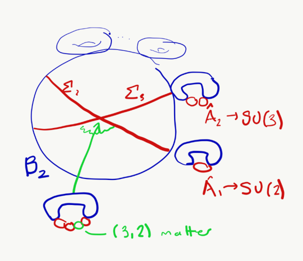
Based on arXiv:2108.07810 by P. Jefferson, WT, A. Turner

F-theory: Nonperturbative formulation of type IIB string theory

Dictionary for geometry $\leftrightarrow$ physics [Vafa, Morrison-Vafa]

$\sim$ compactification of IIB on compact Kähler (non-CY) space B (e.g. $\mathbb{P}^n$)

B_2 (complex surface) $\to$ 6D, $B_3 \to$ 4D.



Elliptic fibration: $\pi : X(CY) \to B$,
$\pi^{-1}(p) \cong T^2$, for general $p \in B$

Fiber singularities $\to$

Gauge group G (codimension 1 in B)

Matter (codimension 2 in B)

Defined by Weierstrass model (fiber τ = 10D IIB axiodilaton)

$$y^2 = x^3 + fx + g, \quad f, g \text{ “functions''} \text{ on } B_2$$

M-theory vs. IIB description

Philosophy of this talk: take IIB description seriously

Most work on F-theory involves explicit resolution of singularities $X \to \tilde{X}$ (i.e. M-theory description). e.g. [Witten, Grimm]

Different resolutions $\to$ different details (e.g. intersection #'s)

Want to identify resolution-independent structure

– Physics must be independent of resolution

– Should be captured by nonperturbative IIB description

– Other recent related work [Grassi/Halverson/Long/Shaneson/Sung, Katz/WT]

– Focus here: structure of intersection theory on singular elliptic CYs

Topology of elliptic Calabi-Yau fourfolds

Divisors: codimension one algebraic 3-folds (7-brane loci)

Shioda-Tate-Wazir:

$$h^{1,1}(X) = h^{1,1}(B) + \text{rk } G + 1$$

Indices:

$D_0 =$ zero section,

$D_\alpha = \pi^* D_\alpha^{(B)}$,

$D_i =$ Cartan generators,

$D_a =$ U(1)'s (Mordell-Weil sections)

Denote collectively by D_I

Nonabelian gauge factors supported on $\Sigma = \sum_\alpha \Sigma^\alpha D_\alpha$

Topology of elliptic Calabi-Yau fourfolds II

Hodge numbers for elliptic CY fourfold

$h^{3,1} =$ # complex structure moduli, $\quad h^{2,1}$ generally 0 or small

$h^{2,2} = 4(h^{1,1} + h^{3,1}) + 44 - 2h^{2,1}, \quad \chi = 6(8 + h^{1,1} + h^{3,1} - h^{2,1})$

For fluxes and chiral matter, we are interested in *vertical* cohomology

$$H_{2,2}^{\text{vert}} = \text{span}_{\mathbb{Z}}(H^{1,1}(X,\mathbb{Z}) \wedge H^{1,1}(X,\mathbb{Z}))$$

Denote $S_{IJ} = D_I \cap D_J$; note, homology relations $\rightarrow$ linear dependencies

Fluxes in $H_{2,2}^{\text{vert}} \rightarrow$ chiral matter

$H^4(X)$ has orthogonal decomposition [Greene/Morrison/Plesser, Braun/Watari]

$$H^4(X,\mathbb{C}) = H^{2,2}_{\text{vert}}(X,\mathbb{C}) \oplus H^{2,2}_{\text{rem}}(X,\mathbb{C}) \oplus H^4_{\text{hor}}(X,\mathbb{C}) \,.$$

$H^4(X,\mathbb{Z})$ has a unimodular intersection pairing

Chiral matter in 4D F-theory models

Flux: $G_{\mathbb{Z}} = G - \frac{c_2(X)}{2} \in H^4(X, \mathbb{Z})$ [Witten]

Satisfies various conditions

SUSY $\Rightarrow G \in H^{2,2}(X, \mathbb{R}) \cap H^4(X, \mathbb{Z}/2), J \wedge G = 0$ [Becker^2, GVW]

Tadpole: $N_{\text{M2}} = \frac{\chi}{24} - \frac{1}{2}\int_X G \wedge G \in \mathbb{Z}_{\geq 0}$ [SVW, DM, DRS]

Poincaré invariance: $\int_{S_{0\alpha}} G = 0\,, \quad \int_{S_{\alpha\beta}} G = 0$

Gauge symmetry preserved: $\int_{S_{i\alpha}} G = 0$ (for E_7 breaking will be $\neq 0$!)

Chiral matter is determined by fluxes, primarily through vertical cycles

Chiral matter: $\chi_{\mathsf{r}} = n_{\mathsf{r}} - n_{\mathsf{r}^*} = \int_{S_{\mathsf{r}}} G \qquad$ (S_{r} a "matter surface")
[Donagi/Wijnholt, Braun/Collinucci/Valandro,
Marsano/Schäfer-Nameki,Krause/Mayrhofer/Weigand, Grimm/Hayashi]

Intersection form on middle cohomology

Previous work on chiral matter in F-theory models used explicit resolutions

Our approach identifies a resolution-independent structure allowing systematic and base-independent analysis for many gauge groups

Basic idea:
M_{IJKL} intersection numbers on CY4 X generally depend on resolution.

Organize as matrix on $H^{\text{vert}}_{2,2}$: $\quad M_{(IJ)(KL)} = M_{IJKL} = S_{IJ} \cdot S_{KL}$.

We then have fluxes $\Theta_{IJ} = \int_{S_{IJ}} G = M_{(IJ)(KL)} \phi^{KL}$,
where $G = \sum_{KL} \phi_{KL} \, \mathrm{PD}(S_{IJ})$.

Removing the null space associated with trivial homology elements,

$$M \to M_{\text{red}} \quad \text{is nondegenerate}$$

Observation/conjecture: M_{red} is resolution independent up to basis
(seen in large classes of examples, general argument with one assumption)

Explicit form of $M_{\rm red}$

Can compute general form of $M_{\rm red}$ for various gauge groups over general bases, using systematic approach to resolution building on earlier work [Esole/Jefferson/Kang]

e.g. simple nonabelian G in basis $S_{0\alpha}, S_{\alpha\beta}, S_{i\alpha}, S_{ij}$

$$M_{\rm red} = \begin{pmatrix} D_{\alpha'} \cdot K \cdot D_\alpha & D_{\alpha'} \cdot D_\alpha \cdot D_\beta & 0 & 0 \\ D_{\alpha'} \cdot D_{\beta'} \cdot D_\alpha & 0 & 0 & * \\ 0 & 0 & -\kappa^{ij} \Sigma \cdot D_\alpha \cdot D_{\alpha'} & * \\ 0 & * & * & * \end{pmatrix} .$$

or after a (non-integral) change of basis

$$U^{\rm t} M_{\rm red} U = \begin{pmatrix} D_{\alpha'} \cdot K \cdot D_\alpha & D_{\alpha'} \cdot D_\alpha \cdot D_\beta & 0 & 0 \\ D_{\alpha'} \cdot D_{\beta'} \cdot D_\alpha & 0 & 0 & 0 \\ 0 & 0 & -\kappa^{ij} \Sigma \cdot D_\alpha \cdot D_{\alpha'} & 0 \\ 0 & 0 & 0 & \frac{M_{\rm phys}}{(\det \kappa)^2} \end{pmatrix} ,$$

where $M_{\rm phys}$ encodes the physics of chiral matter and fluxes.

Example: SU(5) chiral matter (see also [Blumenhagen/Grimm/Jurke/Weigand, Grimm/Krause/Weigand, Marsano/Schafer-Nameki, Grimm/Hayashi])

Can compute from M_{red}

$$\Theta_{33} = \Sigma \cdot K \cdot (6K + 5\Sigma)(\phi^{33} - \phi^{35} - \phi^{44} + \phi^{45})/5\,.$$

Using matter surfaces or cnxn to 3D CS couplings ([Cvetic/Grimm/Klevers])

$$\chi_{\mathbf{5}} = -\Theta_{33} = -\chi_{\mathbf{10}}\,.$$

So we have, where generally m is an integer (exceptions e.g. if $5|K$)

$$\chi_{\mathbf{5}} = \Sigma \cdot K \cdot (6K + 5\Sigma)m\,.$$

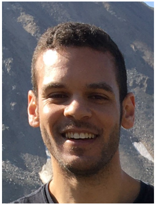
Base-independent formula for chiral multiplicities
($\sim$ [Cvetic/Grassi/Klevers/Piragua] w/ U(1) factors)

For example for $B = \mathbb{P}^3, \Sigma = nH, -K = 4H$,

$$\chi_{\mathbf{5}} = 4(5n - 24)m$$

Some interesting questions regarding quantization remain (see part 3)

2(A). Universal tuned standard model structure in F-theory



Patrick Jefferson

Nikhil Raghuram

Andrew Turner

Based on:

arXiv:1906.11092 by WT, A. Turner

arXiv:1912.10991 by N. Raghuram, WT, A. Turner

arXiv:2201.nnnnn? by P. Jefferson, WT, A. Turner

Generic matter for fixed group G: [WT/Turner]

• Matter in highest dimensional branch of (geometric) moduli space; same in 6D, 4D (least tuning)

• Matches simplest singularities in F-theory

• e.g. $SU(N)$: $\{\square, \begin{smallmatrix}\square\\\square\end{smallmatrix}$, adjoint$\}$

$SU(3) \times SU(2) \times U(1)$: Standard Model matter not generic (e.g. no $(3, 2)_{q \neq 0}$)

$G_{\rm SM} = (SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$: SM matter + several exotics generic

For given G, generic matter typical, anything else fine-tuned

e.g. $SU(N)$ $\square\square$, SU(2) $\square\square\square$ possible "exotic" matter in F-theory [Klevers/Morrison/Raghuram/WT]

Universal G models

For fixed G, matter representations, a *universal G model* is a class of Weierstrass models of full dimensionality (fixed by anomalies in 6D) that geometrically realize G

- Tate models for simple $G = SU(N), E_8, E_7, E_6, F_4, SO(N), G_2, \ldots$
- Morrison-Park model for U(1) with $q = 1, 2$

Universal Weierstrass model for $G_{\rm SM}$ [Raghuram/WT/Turner]

$$\begin{aligned}
f &= -\frac{1}{48}\left[s_6^2 - 4b_1(d_0 s_5 + d_1 s_2)\right]^2 \\
&\quad + \frac{1}{2} b_1 d_0 \left[2b_1\left(d_0 s_1 s_8 + d_1 s_2 s_5 + d_2 s_2^2\right) - s_6(s_2 s_8 + b_1 d_1 s_1)\right], \\
g &= \frac{1}{864}\left[s_6^2 - 4b_1(d_0 s_5 + d_1 s_2)\right]^3 + \frac{1}{4} b_1^2 d_0^2 \left(s_2 s_8 - b_1 d_1 s_1\right)^2 - b_1^3 d_0^2 d_2 \left(s_2^2 s_5 - s_2 s_1 s_6 + b_1 d_0 s_1^2\right) \\
&\quad - \frac{1}{24} b_1 d_0 \left[s_6^2 - 4b_1(d_0 s_5 + d_1 s_2)\right]\left[2b_1\left(d_0 s_1 s_8 + d_1 s_2 s_5 + d_2 s_2^2\right) - s_6(s_2 s_8 + b_1 d_1 s_1)\right].
\end{aligned}$$

(Derived from “unHiggsing” Raghuram’s U(1) $q = 1, 2, 3, 4$ model)

- Includes “F_{11}” $G_{\rm SM}$ models as a special case [Klevers/Mayorga Peña/Oehlmann/Piragua/Reuter]

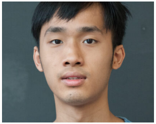
Generic matter for G_{SM} models

	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$(\mathbf{3},\mathbf{1})_{\frac{2}{3}}$	$(\mathbf{3},\mathbf{1})_{-\frac{1}{3}}$	$(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$(\mathbf{1},\mathbf{1})_{1}$	$(\mathbf{3},\mathbf{1})_{-\frac{4}{3}}$	$(\mathbf{1},\mathbf{2})_{\frac{3}{2}}$	$(\mathbf{1},\mathbf{1})_{2}$
(MSSM)	1	-1	-1	-1	1	0	0	0
(exotic 1)	2	-1	-4	-2	0	1	0	1
(exotic 2)	-2	2	2	-1	0	0	1	-1

Analysis: [Jefferson/WT/Turner, to appear]

- Generically get all three families from universal model – no constraints from geometry beyond anomaly cancellation
- Closed form formulae for chiral multiplicities χ_i
- Tuning two discrete parameters gives SM families
- Special case: F_{11} model, recent analysis of 10^{15} 3-generation solutions [Cvetič/Halverson/Lin/(Liu/Tian, Long)]

2(B). Standard model from E_7 breaking in F-theory



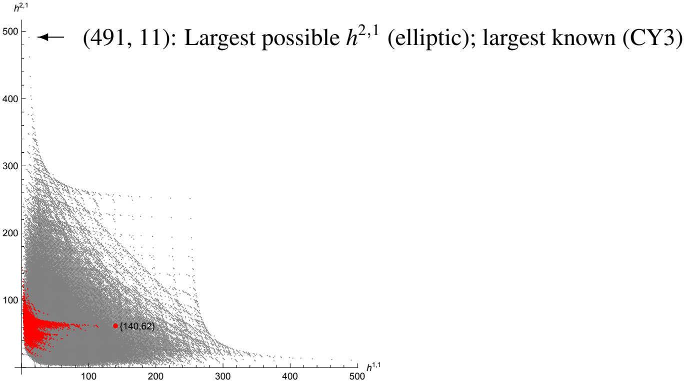
Shing Yan (Kobe) Li

Based on:

arXiv:2112.03947, 22mm.nnnnn by S.Y. Li, WT

Some preliminary global features of the F-theory landscape

Most known Calabi-Yau threefolds and fourfolds are elliptic
(Empirical results, theoretical arguments: [Huang/WT, Anderson/Gao/Gray/Lee])
KS: all but red ones [$\sim$ 30k/400M] admit elliptic/g1 fibration



Set of elliptic Calabi-Yau threefolds bounded, finite, well-described
Similar for CY4 but less complete classification

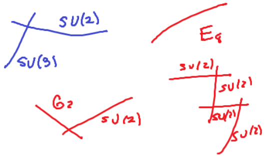
Rigid (non-Higgsable) gauge groups [(Morrison/WT)^2]

In 6D and 4D, most bases force geometrically non-Higgsable G
IIB: 7-branes nucleate on *rigid* loci w/ negative normal bundle

Rigid gauge factors (4D): $SU(2), SU(3), G_2, SO(7), SO(8), F_4, E_6, E_7, E_8$
Note, however, not SU(5)

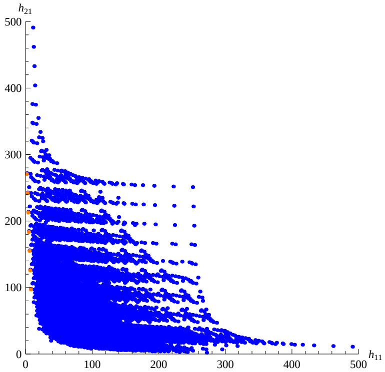
Products of two factors with joint matter (4D):
$G_2 \times SU(2), SO(7) \times SU(2), SU(2) \times SU(2), SU(3) \times SU(2), SU(3) \times SU(3)$

Non-Higgsable clusters only interact through gravity, scalars,
provide natural dark matter candidates



Prevalence of non-Higgsable gauge groups

6D SUGRA/F-theory: One large moduli space of connected branches



[All but orange branches contain NHC's, $\sim$ 61000 toric bases]

Typical $G : E_8^5 \times F_4^6 \times (G2 \times SU(2))^{10}$;

4D: similar story; $\sim 4000/10^{3000}$ (weak Fano) bases lack NHC's

F-theory approaches to the standard model

There are many different ways the standard model may be realized in F-theory

	GUT	$\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$
Tuned G	Tuned GUT (e.g., $\mathrm{SU}(5)$)	Direct tuned G_{SM}
Non-Higgsable G	Non-Higgsable GUT (e.g., $\mathrm{E}_6, \mathrm{E}_7$)	Non-Higgsable G_{SM}

- Previous discussion: direct tuned
- Much work: tuned GUT e.g. $\mathrm{SU}(5)$ [Beasley/Heckman/Vafa, Donagi-Wijnholt]

Tuned models are rare in landscape, however: require tuning many moduli, many bases will not support

- $\mathrm{SU}(3) \times \mathrm{SU}(2)$ can be geometrically non-Higgsable in 4D [Grassi/Halverson/Shaneson/WT]; $\mathrm{U}(1)$ factor difficult however to integrate

Most natural approach: non-Higgsable GUT

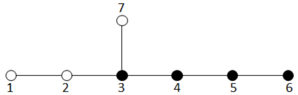
Next: breaking $E_7 \to G_{\mathrm{SM}}$ with fluxes

Breaking $E_7 \to G_{\text{SM}}$ [SY (Kobe) Li/WT, arXiv:2112.03947]

Recall

$$\Theta_{IJ} = \int_{S_{IJ}} G = M_{(IJ)(KL)}\phi^{KL}\,.$$

When $\Theta_{i\alpha} \neq 0$, breaks Cartan generator i; $\sum_i c_i \Theta_{i\alpha} = 0 \forall \alpha$ preserves U(1), etc.



Can choose fluxes to break $i = 3, 4, 5, 6$ for any geometric E_7, leaving $SU(3) \times SU(2)$

Note: this realization of $SU(3) \times SU(2)$ is unique up to E_7 automorphism

Depending on fluxes, preserve different $U(1)$ factors, different spectra

– Many $SU(3) \times SU(2) \times U(1)$ breakings, but most have exotics

Intermediate SU(5) and remainder hypercharge flux breaking

To avoid exotics, any appropriate $U(1) \to SU(5)$ enhancement!
(flux vanishes on an additional $\mathbb{P}^1$; equivalent to $\Theta_{3\alpha} = 0$)

Proceed in two steps: 1) Vertical flux breaking $E_7 \to SU(5)$,
2) Remainder flux breaking $SU(5) \to G_{\rm SM}$
($\sim$ [Beasley/Heckman/Vafa, Donagi-Wijnholt, Blumenhagen/Grimm/Jurke/Weigand, Marsano/Saulina/Schafer-Nameki, Grimm/Krause/Weigand, ...])

Remainder flux:

$$G_4^{\rm rem} = [D_Y|_{C_{\rm rem}}] \, ,$$

where $D_Y = 2D_1 + 4D_2 + 6D_3 + 3D_7$ generates hypercharge.

$C_{\rm rem}$ is a curve on Σ, homologically trivial in B. Such curves exist on typical non-toric bases [Braun/Collinucci/Valandro]

Matter content with this breaking contains only SM family

$$(\mathbf{3},\mathbf{2})_{1/6}\, , \quad (\mathbf{3},\mathbf{1})_{2/3}\, , \quad (\mathbf{3},\mathbf{1})_{-1/3}\, , \quad (\mathbf{1},\mathbf{2})_{1/2}\, , \quad (\mathbf{1},\mathbf{1})_{1}\, ,$$

arising from (non-chiral) E_7 representations **56** and **133**.

A simple example (chiral multiplicity for SU(5) only)

We consider the base B a $\mathbb{P}^1$ bundle over Hirzebruch $\mathbb{F}_1$,
Σ an $\mathbb{F}_1$ section with normal bundle $N_\Sigma = -8S - 7F$
(S, F generate divisors of B with $S \cdot S = -1, S \cdot F = 1, F \cdot F = 0$)

$\Rightarrow$ rigid E_7 factor on Σ

To solve the flux constraints in the Kähler cone we need:

$0 > \phi_{iS}/\phi_{iF} = n_S/n_F \neq \infty$ identical for all i

We then have:

$\chi_{(\mathbf{3},\mathbf{2})_{1/6}} = 7n_S + 4n_F, \quad (\phi_{1S}, \phi_{2S}, \phi_{3S}, \phi_{4S}, \phi_{7S}) = (2,4,6,5,3)n_S \;\; (+S \to F)$

From $\chi(X) = 51096, h^{2,2}(X) = 34076 \gg \chi(X)/24$, a random flux typically has most entries 0 and small nonzero values.

Minimal solution:

$n_S = -n_F = \pm 1 \Rightarrow$ Number of generations is ± 3

While this is just one example, others have other values, this local structure is ubiquitous in the landscape. Expect similar for geometries with rem flux.

Features of $E_7 \to G_{\mathrm{SM}}$ flux construction

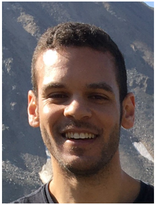
- Ubiquitous/natural: construction is possible on typical bases
  estimate 18% of base threefolds have rigid E_7 [WT/Wang]
- Flux breaking of GUT E_7 without its own chiral matter
- No chiral exotics for certain breaking pattern with intermediate SU(5)
- Chiral multiplicity is naturally small.

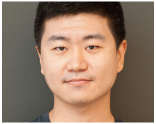
- Similar construction possible for E_6, more complicated
- Does not work for E_8, but maybe from SCFT matter? [Tian/Wang]

More in upcoming longer paper …

3. Mirror symmetry in 4D F-theory models


Yu-Chien Huang


Patrick Jefferson


Manki Kim


Paul Oehlmann

Based on:

arXiv:1811.04947 by Y-C Huang, WT

arXiv:22mm.nnnnn? by P. Oehlmann, WT

arXiv:22mm.nnnnn? by P. Jefferson, M. Kim, WT

Mirror symmetry factorizes for many toric hypersurfaces! [Huang/WT]

Toric hypersurface associated with reflexive polytope ∇; mirror dual Δ.

Elliptic if $\nabla_2 \subset \nabla$ is reflexive 2-polytope.

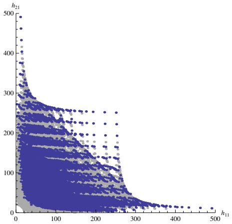
If $F = \nabla_2 \subset \nabla$ is a slice and $\tilde{F} = \Delta_2 \subset \Delta$ is also a slice
$\Rightarrow$ Mirror symmetry factorizes

Simplest cases: Standard stacking on $\mathbb{P}^{2,3,1} \leftrightarrow$ Tate form Weierstrass model

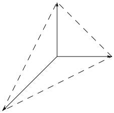
Mirror of generic elliptic fibration over B = ef over $\tilde{B}$ (may be tuned):

$$B \to \tilde{B} \sim \Sigma(-6K_B), \nabla_2 = \Delta_2 = \mathbb{P}^{2,3,1}$$

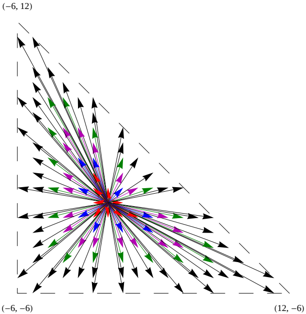
(65k examples in KS database)



Example: generic elliptic fibration on $\mathbb{P}^2$ (2, 272)



Hodge numbers (2, 272)



Hodge numbers (272, 2)
(toric rays: $\vec{w} \cdot \vec{v} \geq -6$, $\forall \vec{v} \in \Sigma_B$, $\vec{w}$ primitive)

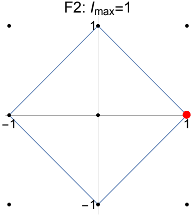
$$h^{1,1}(B) = 1$$
$$G = 1$$
$$h^{1,1}(X) = h^{1,1}(B) + \text{rk}\, G + 1 = 2$$
$$h^{2,1}(X) = 301 - 29h^{1,1}(B) - \dim M_{nh} = 272$$

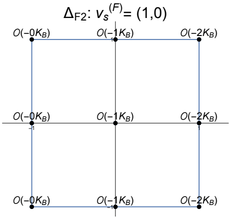
$$h^{1,1}(B) = 106 + 3 = 109$$
$$G = E_8{}^9 \times F_4{}^9 \times (G_2 \times SU(2))^{18}$$
$$h^{1,1}(X) = h^{1,1}(B) + \text{rk}\, G + 1 = 272$$
$$h^{2,1}(X) = 301 - 29h^{1,1}(B) + \dim G - \dim M_{nh} = 2$$

Factorized mirror symmetry: more general structures [Oehlmann/WT, to appear]

- Also works for "tuned" Tate models $\leftrightarrow$ reduction on Δ
- Works for other fibers, bundle structures

e.g. $B = \mathbb{P}^2, F = F_2$; base stacked over vertex: $H = (4, 94)$
$\tilde{B} \sim -2K_B, \tilde{F} = F_{15}$; $H = (94, 4)$





(mirror symmetry of fibers:[Klevers/Mayorga Pena/Oehlmann/Piragua/Reuter])

- Many interesting features, allows exploration of e.g. Higgsing transitions on superconformal matter through mirror

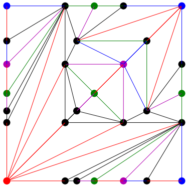
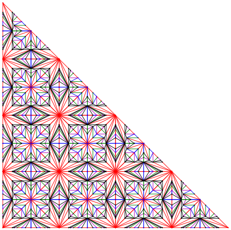
Factorize mirror symmetry for CY fourfolds: similar story

Example: $B = \mathbb{P}^3$ standard stacking ($F = \mathbb{P}^{2,3,1} = F_{10}$)

Rays in $\tilde{B}$: primitive lattice points in tetrahedron:
w/vertices (-6, -6, -6), (18, -6, -6), (-6, 18, -6), (-6, -6, 18)

$G = E_8^{34} \times F_4^{96} \times G_2^{256} \times SU(2)^{384}$

- (Exponentially) many triangulations; construction from (projected) tiling


$\rightarrow$


[Jefferson/Kim/WT]

- Note: common endpoint from random blow-up sequence [WT/Wang]

Combining factorization of mirror symmetry on CY fourfolds with structure of M_{red} allows computation of full unimodular intersection form on $H_4(X, \mathbb{Z})$ [Jefferson/Kim/WT work in progress]

Example: $B = \mathbb{P}^3$

$h^{1,1}(X) = 2, h^{3,1}(X) = 3878$

Mirror symmetry: $h^{1,1}(Y) \leftrightarrow h^{3,1}(X)$

With 2306 toric rays and 22 E_8 factors with (4, 6) loci blown up non-torically,

$$\begin{aligned} h^{1,1}(Y) &= h^{1,1}(\tilde{B}) + \text{rk}\tilde{G} + 1 \\ &= 2303 + 22 + (34 \times 8 + 96 \times 4 + 256 \times 2 + 384) + 1 \\ &= 3878 \end{aligned}$$

Expect that full intersection form on $H_4(X, \mathbb{Z})$ includes

$$M_{\text{red}}(X, \mathbb{Z}) \oplus M_{\text{red}}(Y, \mathbb{Z})$$

since $H_{2,2}^{\text{vert}} \leftrightarrow H_{2,2}^{\text{hor}}$ [Braun/Watari], here $H_{2,2}^{\text{rem}} = 0$.

Expect $M_{\rm red}(X,\mathbb{Z}) \oplus M_{\rm red}(Y,\mathbb{Z})$ is unimodular or has unimodular overlattice

In the example $X = \mathbb{P}^3$, there is no gauge group so

$$M_{\rm red}(X,\mathbb{Z}) = \begin{pmatrix} K \cdot D_{\alpha'} \cdot D_\alpha & D_\alpha \cdot D_\beta \cdot D'_\alpha \\ D_\alpha \cdot D'_\alpha \cdot D'_\beta & 0 \end{pmatrix} = \begin{pmatrix} -4 & 1 \\ 1 & 0 \end{pmatrix}$$

This is unimodular so expect $M_{\rm red}(Y,\mathbb{Z})$ also unimodular.

$$M_{\rm red}(Y,\mathbb{Z}) \sim \begin{pmatrix} D_{\alpha'} \cdot K \cdot D_\alpha & D_{\alpha'} \cdot D_\alpha \cdot D_\beta & 0 & 0 \\ D_{\alpha'} \cdot D_{\beta'} \cdot D_\alpha & 0 & 0 & * \\ 0 & 0 & -\kappa^{ij}\Sigma \cdot D_\alpha \cdot D_{\alpha'} & * \\ 0 & * & * & * \end{pmatrix}.$$

Upper left 2×2 unimodular by Poincare duality, toric curves span Chow ring

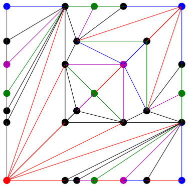
E_8 factors unimodular, $F_4 \to$ overlattice/extra surfaces, $G_2 \times SU(2)$ extra surfaces.

Some technical details but unimodular structure appears to arise for a general class of bases. Gluing from $G \to G$, or $[G, E_8]$ ("E_8 rule" [Berglund/Mayr])

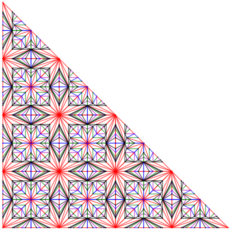
Example computation: $h^{\text{vert}}_{2,2}(Y)$

Counting independent contributions from $S_{0\alpha}, S_{\alpha\beta}, S_{i\alpha}$,

$|S_{0\alpha}| = |S_{\alpha\beta}| = h^{1,1}(\tilde{B}) = 2325 \rightarrow 4650$



$\rightarrow$

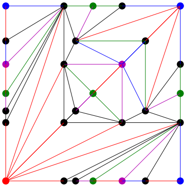


$|S_{i\alpha}| = \sum_i \text{rk}\, \tilde{G}_i(h^{1,1}(\Sigma_i)) = 8 \times (30 \times 22 + 4 \times 16) + 4 \times (32 \times 14 + 64 \times 2) + 2(128 \times 4 + 128 \times 2) + (384 \times 2) \rightarrow 10400$

From mirror symmetry, $H^{\text{vert}}_{2,2}(Y) = 15562 = 10400 + 4650 + 512$

Remaining 512 surfaces: 256 from $G_2 \times SU(2)$ clusters, 256 from F_4 factors with codimension 3 (4, 6) loci

Example: extra surfaces from $G_2 \times SU(2)$ clusters



$$\rightarrow \begin{pmatrix} K D_\alpha^2 & D_\alpha^3 & 0 & 0 & 0 & 0 \\ D_\alpha^3 & 0 & 0 & 0 & 0 & -2 D_\alpha^2 \Sigma_2 \\ 0 & 0 & -6 D_\alpha^2 \Sigma_1 & 3 D_\alpha^2 \Sigma_1 & 0 & -2 D_\alpha \Sigma_1 \Sigma_2 \\ 0 & 0 & 3 D_\alpha^2 \Sigma_1 & -2 D_\alpha^2 \Sigma_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 D_\alpha^2 \Sigma_2 & D_\alpha (4K + 3\Sigma_1 - 2\Sigma_2) \Sigma_2 \\ 0 & -2 D_\alpha^2 \Sigma_2 & -2 D_\alpha \Sigma_1 \Sigma_2 & 0 & D_\alpha (4K + 3\Sigma_1 - 2\Sigma_2) \Sigma_2 & -\Sigma_2 \left(8K^2 + 5\Sigma_1^2 + \Sigma_1 (13K - 3\Sigma_2) + 2\Sigma_2 (-2K + \Sigma_2)\right) \end{pmatrix}$$

$G_2, SU(2)$ factors on e.g. local Hirzebruch F_{12}, F_6 surfaces for case on LHS

Can compute explicitly ... [Work in progress],
expect $\det = K^2$, necessary for overlattice

Unimodular structure: overlattices

Some components of $H^{2,2}_{\text{vert}}(Y)$ not immediately unimodular: need overlattice

Example: F_4 inverse killing form

$$\kappa(F_4) = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -2 & 0 \\ 0 & -2 & 4 & -2 \\ 0 & 0 & -2 & 4 \end{bmatrix}$$

This is not unimodular: $\det \kappa(F_4) = 4$

But adding an additional lattice vector
$(0, 0, 1/2, 1/2) = (S_{\Sigma 3} \cap S_{\Sigma 3} + S_{\Sigma 4} \cap S_{\Sigma 4})/2 \rightarrow$ unimodular!

Gives proper quantization for integral lattice.

Presence of extra vectors guaranteed by unimodularity of $H_4(X, \mathbb{Z})$
Confirmation from other approaches–work in progress.

Computation of full $H_4(X, \mathbb{Z})$: further issues

– For $B = \mathbb{P}^3$ example, $H_4(X, \mathbb{Z}) = M_{\text{red}}(X, \mathbb{Z}) \oplus M_{\text{red}}(Y, \mathbb{Z})$, both terms must be unimodular

– More generally $M_{\text{red}}(X, \mathbb{Z})$ not unimodular,
from non-Higgsable + tuned G blocks
Expect complement has G or $[E_8, G]$ (observed in toric duals)

– Also, generally nontrivial $H^{\text{rem}}_{2,2}$, need to compute intersection form on this

Conclusions

• New general approach to understanding resolution-independent intersection form on $H_{2,2}^{\text{vert}}$, key for understanding flux compactifications and chiral matter

• General formulae for chiral matter including for universal G_{SM} model; in all cases independent families of chiral matter only constrained by anomalies

• New approach to realizing Standard Model gauge group and chiral matter with 3 generations and no exotics from flux breaking of
$E_7 \to SU(5) \to (SU(3) \times SU(2) \times U(1))/\mathbb{Z}_6$

• Structure of intersection form M_{red} allows computation of full integer intersection form on $H_{2,2}(X, \mathbb{Z})$ using mirror symmetry