EXERCISES

We repeat the discussion of $SU(N_c)$ gauge theories for $SP(N_c)$ (Problems 1,3,5) and $SO(N_c)$ (Problems 2,4,6).

Problem 1. Consider an $SP(N_c)$ gauge theory with $2N_f$ chiral superfields in the fundamental ($2N_c$ dimensional) representation.

(a) Find the moduli space of classical ground states and parametrize them by gauge invariants. Write explicitly the constraints when they exist. What is the unbroken gauge group at generic points on the moduli space?

(b) Find the symmetries of the quantum theory. What is the most general invariant superpotential which can be generated? When can we conclude that no superpotential is generated?

(c) What happens if the number of chiral superfields is odd?

Problem 2. Repeat the analysis of Problem 1 for an $SO(N_c)$ gauge theory with N_f chiral superfields in the fundamental (N_c dimensional) representation.

Problem 3. We continue our study of $SP(N_c)$ gauge theory with $2N_f$ chiral superfields in the fundamental ($2N_c$ dimensional) representation from Problem 1.

(a) Find when instantons can generate the superpotential, and check the necessary conditions we discussed in $SU(N_c)$. Use the results for $SU(N_c)$ to show that the coefficient is not zero in this case.

(b) Use (a) to derive the superpotential for smaller values of N_f. Show that in the pure gauge $SP(N_c)$ theory $\langle\lambda\lambda\rangle \neq 0$.

(c) Describe the moduli space of quantum vacua for one more flavor than in (a). Find the most symmetric points on the moduli space, and identify the spectrum of massless particles there. Check the 't Hooft anomaly matching conditions there.

(d) Add one more flavor to (c). What is the moduli space of quantum vacua? What is the low energy field theory? Find the most symmetric point in the moduli space and the spectrum of massless particles there. Check the 't Hooft anomaly matching conditions there.

Problem 4. We continue our study of $SO(N_c)$ gauge theory with N_f chiral superfields in the fundamental (N_c dimensional) representation from Problem 2. For $N_c = 4$ limit the analysis to equal gauge coupling constants for the two $SU(2)$ subgroups.

(a) Use these theories and the information in $SU(N_c)$ theories to show that $\langle\lambda\lambda\rangle \neq 0$ in the pure gauge $SO(N_c)$ theories. Derive the superpotential for $SO(N_c)$ with $N_f \leq N_c - 5$.

(b) Find the effect of gluino condensation in the unbroken gauge group for $N_f = N_c - 4$. How many branches are there? Find the spectrum of massless particles in all supersymmetric vacua.

(c) Use (b) to find the superpotential for $N_f = N_c - 3$. What is the spectrum of massless particles in the supersymmetric vacua? Check the 't Hooft anomaly matching conditions. Find the physical origin of the various terms in the superpotential.

(d) Use (c) to determine the coefficients of all instanton effects we discussed and the value of $\langle\lambda\lambda\rangle$ in the pure gauge theories using information involving one loop computations only.

(e) Find the moduli space of vacua for $N_f = N_c - 2$. At the generic point on this space the spectrum includes a massless photon. Limit your analysis to $N_c \geq 4$. Find the elliptic curve which determines its coupling constant, τ_{eff}. Note that this problem has only $N = 1$ supersymmetry but τ_{eff} is still holomorphic, and therefore can be determined. Identify the singularities on the moduli space, and find the spectrum of massless particles there. Check the 't Hooft anomaly matching conditions. Show that upon adding a mass to one of the flavors the theory goes over to the one found in (c). Compare with the theory with $N_c = 3$, which has $N = 2$ supersymmetry.

Problem 5. We continue our study of $SP(N_c)$ gauge theory with $2N_f$ chiral superfields in the fundamental ($2N_c$ dimensional) representation.

(a) Find the dual gauge group. When is it UV free and when is it IR free?

(b) Find the map of the chiral operators. Show that their dimensions satisfy the unitarity bound.

(c) Check the 't Hooft anomaly matching conditions.

(d) Check the flat directions.

(e) Check the mass deformations.

Problem 6. We continue our study of $SO(N_c)$ gauge theory with N_f chiral superfields in the fundamental (N_c dimensional) representation. For $N_c = 4$, limit the analysis to equal gauge coupling constants for the two $SU(2)$ subgroups. Repeat all the questions from Problem 5 for this case. Note the special peculiarities when the dual gauge group is $SO(3)$.