Term 2, Problem Set 3

(1) Let $G(k,n)$ be the Grassmannian of complex k planes in $\mathbf{C}^n$. Consider the two-dimensional sigma model with $G(k,n)$ as target space. Determine the large n behavior for fixed k. Be sure to include the "theta angle" related to $H^2(G(k,n),\mathbf{Z}) = \mathbf{Z}$. Answer the usual questions: is there a mass gap? Are symmetries broken? What are the quantum numbers of the lightest particles?

(2) Now do the same for $G_{\mathbf{R}}(k,n)$, the Grassmannian of real k planes in $\mathbf{R}^n$. Note that the "theta angle" is now $\mathbf{Z}/2\mathbf{Z}$-valued as $H^2(G_{\mathbf{R}}(k,n),\mathbf{Z}) \cong \mathbf{Z}/2\mathbf{Z}$.

In this case, to understand the case that the theta angle is non-zero, you might find it helpful to look at my paper "θ Vacua In Two-Dimensional Quantum Chromodynamics," Il Nuovo Cimento **51 A** (1979) 325. (A few copies will be provided.)