STRING THEORY

Problem Set # 3

February 6, 1997

Transition amplitudes of a single point particle in Quantum Field Theory may be described by summation over the space of paths, in complete analogy with the summation over surfaces that we prescribed for transition amplitudes of strings. In this problem set, we propose to show that certain point particle amplitudes, prescribed this way, precisely agree with those obtained from corresponding Feynman diagrams.

Let $M = \mathbf{R}^D$ be flat Euclidean space-time, and define the world-line action for a single point particle of electric charge e in the presence of a fixed external U(1) gauge field A by

$$S[x,g,A] = \frac{1}{2}\int_0^1 d\tau \left\{ g(\tau)^{-1}\dot{x}\cdot\dot{x} + g(\tau)m_0^2 \right\} + ie\int_0^1 d\tau \dot{x}\cdot A(x) \qquad (1)$$

We define amplitudes for transition between points $y,\ y' \in M$ by

$$A(y,y') = \int_{Met([0,1])} Dg \frac{1}{\mathcal{N}(g)} \int_{Map([0,1];M)} Dx e^{-S}$$

The measures Dg and Dx are defined with respect to the Diff([0,1]) invariant L^2 norms on δg and δx, and $\mathcal{N}(g)$ is formally equal to the volume of Diff([0,1]).

1) Derive the infinitesimal transformations under Diff([0,1]) (generated by a vector field v) of g and x, verify that S is invariant and find the Diff([0,1]) invariant L^2 norms of δg and δx.

2) Derive an orthogonal decomposition of an arbitrary δg into infinitesimal diffeomorphisms and a change in length $L = \int_0^1 d\tau g(\tau)$.

3) Use the above decomposition to factorize the measure Dg into $Dv\ dL$, and compute the associated Jacobian.

4) Show that $A(y,y')$ coincides with the Green function for a scalar field of mass m in the presence of an Abelian gauge field A. (The relation between m_0 and m may depend upon the regularization scheme used to calculate the Jacobian in 3.)

5) Generalize the above prescription and adapt it to the calculation of one loop amplitudes for charged scalar particles in the presence of an external U(1) gauge field.

6) Using 5), show that the one loop amplitudes obtained this way agree with those derived from Feynman diagrams, in a perturbation expansion in powers of A. (You may restrict attention to the first non-trivial order in A.) Show that the length parameters (L, introduced in 2)) of the worldlines between successive insertions of A in this expansion are related to the Feynman parameters used to evaluate the corresponding Feynman diagrams in momentum space.