STRING THEORY

Problem Set # 6

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In this problem set, we investigate a number of properties of the combined matter (for flat space-time of dimension D) and ghost system. Recall that the worldsheet action in locally flat complex coordinates z and $\bar z$ is given by

$$S = \frac{1}{2\pi}\int_\Sigma d^2z\big(\frac{1}{2}\partial x\cdot\bar\partial x + b\bar\partial c + \bar b\partial\bar c\big).$$

The matter stress tensor is denoted by $T^{(x)}(z)$ and the quantum ghost stress tensor is given by

$$T^{(bc)}(z) = \lim_{w\to z}\big(-\partial b(z)c(w) - 2b(z)\partial c(w) + \frac{1}{(z-w)^2}\big).$$

The classical BRST current is given by

$$j^B(z) = c(z)\big(T^{(x)}(z) + \frac{1}{2}T^{(gh)}(z)\big) + \frac{3}{2}\partial^2 c(z).$$

Problem 1

a) Give a suitable definition of the quantum BRST current, by subtracting OPE singularities.

b) Compute the OPE of j^B with b, c and x, and deduce the (anti-) commutators of the BRST operator

$$Q^B = \oint\frac{dz}{2\pi i}j^B_z$$

with these fields.

c) Show that the resulting transformation laws on the fields b, c and x give rise to a classical symmetry of S for any dimension D of space-time, and that j^B is related to the Noether current of this symmetry.

d) Show that Q^B is nilpotent as a quantum mechanical operator if and only if $D=26$, using your favorite method. (Hint : It is easy to show that when $D\neq 26$, Q^B cannot be nilpotent, by using the relation between the full stress tensor $T = T^{(x)} + T^{(bc)}$ and the anti-commutator of Q^B with b.)

Problem 2

Henceforth, set $D = 26$, and consider correlation functions of vertex operators V_i, including ghosts :

$$\langle V_1 \cdots V_N \rangle = \int Dx \int D(b\bar{b})D(c\bar{c})V_1 \cdots V_N \prod_{j=1}^{\nu} |(\mu_j, b)|^2 e^{-S}$$

For simplicity, we restrict to surfaces Σ of genus $h > 1$; μ_j is a basis of Beltrami differentials, associated with moduli coordinates m_j, and $\nu = 3h - 3$.

Assume that V_i are physical vertex operators, so that V_i commutes with the BRST operator Q^B. Now assume that one vertex operator, say V_N, is BRST exact, i.e.

$$V_N = \int_\Sigma d^2 w \oint_C \frac{dz}{2\pi i} j^B(z) W_N(w)$$

for *some* operator W_N, which is not necessarily a physical vertex operator. The curve C is a simple contour around the point w. By deforming the contour C, show that $\langle V_1 \cdots V_N \rangle$ is a total derivative in the moduli m_j.