STRING THEORY

Problem Set # 7

March 13, 1997

Problem 1

Let M be a Riemannian manifold with local coordinates x^μ, metric $G = G_{\mu\nu} dx^\mu \otimes dx^\nu$, two form $B = B_{\mu\nu} dx^\mu \wedge dx^\nu$, and scalar field Φ, and let l be a real positive parameter. The generalized non-linear sigma model action on a surface Σ, with metric g, Gaussian curvature R_g, volume form $d\mu_g$, and anti-symmetric tensor ϵ^{mn}, is given by

$$S[x;g] = \frac{1}{8\pi l^2} \int_\Sigma d\mu_g \left[g^{mn} G_{\mu\nu} + \epsilon^{mn} B_{\mu\nu} \right] \partial_m x^\mu \partial_n x^\nu + \frac{1}{2\pi} \int_\Sigma d\mu_g R_g \Phi$$

a) Derive the form of the vector field $V^\mu(x)$ in TM, such that the infinitesimal Weyl transformation $\delta g = 2\delta\sigma g$, combined with the transformation $\delta x^\mu = l^2 \delta\sigma V^\mu(x)$ yields a trace of the stress tensor $T_{mn} g^{mn}$ which is of the same form as the Lagrangian density of $S[x;g]$, i.e. such that

$$\delta S[x;g] = -\frac{1}{2\pi} \int_\Sigma d\mu_g \left[g^{mn} \beta^G_{\mu\nu} + \epsilon^{mn} \beta^B_{\mu\nu} \right] \partial_m x^\mu \partial_n x^\nu + \frac{1}{2\pi} \int_\Sigma d\mu_g R_g \beta^\Phi$$

b) Derive the resulting expressions for β^G, β^B and β^Φ. (These are the tree level contributions to the β-functions defined for Weyl transformations.)

Problem 2

Let ϕ be a real scalar field on a Riemann surface Σ, with action

$$S[\phi;g] = \frac{1}{8\pi} \int_\Sigma d\mu_g \left[g^{mn} \partial_m \phi \partial_n \phi + m^2 \phi^2 \right]$$

where m is an IR regulator mass, which we let to zero whenever the limit is well-defined.
a) Use dimensional regularization to derive the Weyl transformation properties of the Green function $G(\xi, \xi') = \langle \phi(\xi) \phi(\xi') \rangle$ and its first and second derivatives, evaluated on the diagonal (i.e. at coincident points).
b) Repeat the calculation of a), but now with a heat-kernel regularization of the Green function. Compare your result with that of a).