STRING THEORY

Problem Set # 10

April 10, 1997

In the following problems, we shall derive certain properties, under so-called T-duality, of closed oriented string theories, compactified in one dimension on a circle of radius R.

Problem 1

Consider the closed oriented bosonic string on the space-time manifold $M = \mathbf{R}^{25} \times S^1$, with Minkowskian metric $\eta = \mathrm{diag}(- + \cdots +)$ and with S^1 of radius $R > 0$ (lying in a space direction of η).

a) Obtain the mode expansion of the most general solution to the free theory on a worldsheet with the topology of an annulus. (Make sure to include the effects of the fact that the space-time manifold M is not simply connected.)

b) Show that the states of the theory for radius R are in one to one correspondence with the states for the theory with radius $R' = b/R$ for some (R-independent) constant $b > 0$. This relation is usually called T-duality.

c) Show that transition amplitudes for the theory with radius R and R' – as defined in b) – are similarly related. One says that the bosonic closed oriented string is self-dual under T-duality.

Problem 2

Consider now the Type IIA and Type IIB closed oriented superstring theories on the space-time manifold $M = \mathbf{R}^9 \times S^1$, with Minkowskian metric $\eta = \mathrm{diag}(- + \cdots +)$ and with S^1 of radius $R > 0$ (lying in a space direction of η).

a) Begin by taking $R = \infty$ (the original superstring theories) and show that the massive spectra of the Type IIA and Type IIB theories are identical.

b) Now, for finite R, obtain the mode expansion of the most general solution to the free theories on a worldsheet with the toplogy of an annulus.

c) Show that the states which are annihilated by all the oscillator annihilation operators of level > 0 (these are the pure momentum and winding states) for the Type IIA string at radius R are in one to one correspondence with states for the Type IIB string at radius $\tilde{R} = c/R$ for some sonstant c. (You will have to explain how the relative chirality of left and right movers can be changed upon $R \to \tilde{R}$.)

d) Generalize the argument of c) to show that the full spectra of the Type IIA superstring at radius R and of the Type IIB superstring at radius $\tilde{R}$ are identical. One says that the Type IIA and Type IIB superstrings are mapped into one another under T-duality.

Problem 3

Finally, consider the heterotic strings with gauge groups $E_8 \times E_8$ and $Spin(32)/Z_2$, on the space-time manifold $M = \mathbf{R}^9 \times S^1$, with Minkowskian metric $\eta = \mathrm{diag}(- + \cdots +)$ and with S^1 of radius $R > 0$ (lying in a space direction of η).

The circle compactification now allows for a non-trivial holonomy of the gauge field along S^1 – in physics language : a non-zero value for the Wilson loop around S^1. We assume that this holonomy element is such that it breaks both $E_8 \times E_8$ and $Spin(32)/Z_2$ down to $SO(16) \times SO(16)$.

Show that the corresponding $SO(16) \times SO(16)$ theories are again related to one another upon $R \to \tilde{R}$, where $\tilde{R}$ was defined in Problem 2, c. One says that the two heterotic theories are mapped into one another under T-duality.