Elizabeth E. Leyton Chisholm

Research Statement

My research interests lie in an area of mathematics called geometric group theory. Geometric group theory uses geometry and topology to study the algebraic structure of groups. My work in particular relies upon linear algebra, group theory, topology and geometry to study the structure of the braid groups. My dissertation describes how braid groups act on euclidean simplices by reshaping them in space. In particular, my work describes a geometric interpretation of the Lawrence Krammer Bigelow represenation of the braid groups. I presented this work at an international conference on Configuration Space in Cortona, Italy this past summer and submitted a paper for publication in the Conference Proceedings, arXiv 1411.1208.

The results of my work have the potential to be extended to other types of groups, particularly Artin groups of finite type. Additionally, work that went into my thesis has the capacity to lead to a new proof that the braid groups are linear which may be extended to Euclidean Artin groups. In particular, my research has heavily relied on computer programming to discover not only the results of my dissertation but also helped make progress toward these goals. These research projects would be accessible to undergraduates because they rely on topics they are familiar with, such as linear algebra, group theory, and computer programming. I am excited about the future potential of my research and I look forward to working with students to realize these possibilities.

My research plan has two main tracks. First, I wish to extend the results of my dissertation to other groups, specifically Artin groups of finite type. Braid groups are a specific case of a general class of groups called Artin groups. The Lawrence Krammer Bigelow representation extends to a linear representation of Artin groups of finite type [CW02]. A defining geometric structure for the Artin groups of finite type are root systems. The plan is to introduce a geometric interpretation of the representation in this situation by describing a reshaping of the root systems.

Second, I hope to use the ideas developed in my dissertation to achieve the original goal of my research. The Lawrence Krammer Bigelow representation extends to another class of Artin groups, the Euclidean Artin groups. It is unknown whether this representation is faithful. The proofs of linearity of the braid groups in [Big01] and [Kra02] use the standard, as opposed to the dual, Garside structure that I use in my work. A new proof that braid groups are linear based on the dual Garside structure has the potential to be extended to Euclidean Artin groups. My initial plan was to develop a new proof that the braid groups are linear using a specialization of the Lawrence Krammer Bigelow representation. While working on this project it became apparent that this representation is not faithful. However, the discoveries in my dissertation make progress towards a solution using the original Lawrence Krammer Bigelow representation.

Computer programming was critical in developing the results of my dissertation. I computed the matrices for the specialized representation and analyzed them using code in Sage. In particular, the main results of my dissertation were made apparent by this code. Modifying this code to examine the original Lawrence Krammer Bigelow representation has led to some promising discoveries that may be used to prove linearity with the dual Garside structure.

Each of these research tracks has enormous potential to involve students. Both involve investigating properties of representations using specific groups, thus providing concrete examples for undergraduates to investigate. Students would utilize familiar topics in these projects, such as linear algebra and group theory, and give them valuable experience in computer programming. Student projects would not only be excellent research experience for students, but would also greatly contribute to the success of my research plan.

References

[Big01] Stephen J. Bigelow, *Braid groups are linear*, J. Amer. Math. Soc. **14** (2001), no. 2, 471–486 (electronic). MR 1815219 (2002a:20043)

[CW02] Arjeh M. Cohen and David B. Wales, *Linearity of Artin groups of finite type*, Israel J. Math. **131** (2002), 101–123. MR 1942303 (2003j:20062)

[Kra02] Daan Krammer, *Braid groups are linear*, Ann. of Math. (2) **155** (2002), no. 1, 131–156. MR 1888796 (2003c:20040)