The Disambiguated Temperley-Lieb Algebra

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UC Santa Barbara

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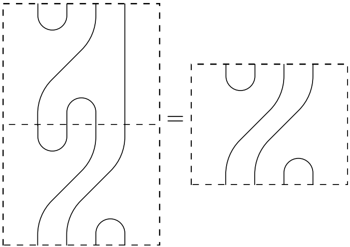

$\in \mathcal{TL}_4$

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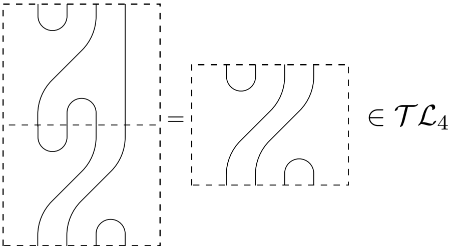

- These vector spaces assemble together into a planar algebra

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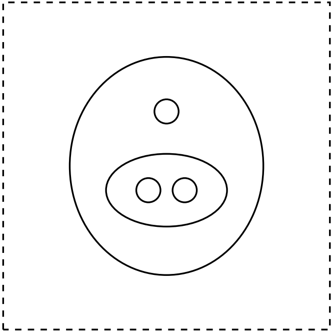

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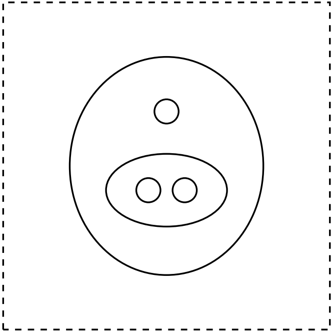

$$\rightarrow \quad \sum_{\text{states}} \prod_{\text{regions}} \#$$

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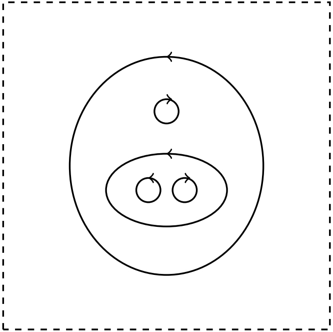

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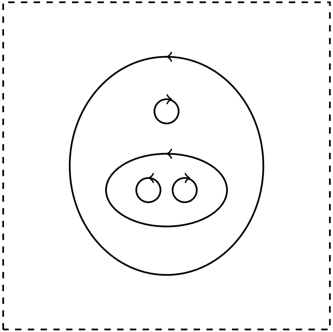

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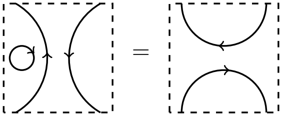

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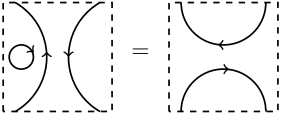

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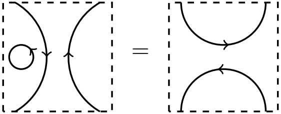

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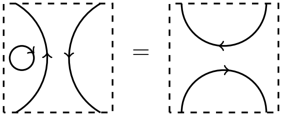

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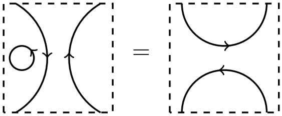
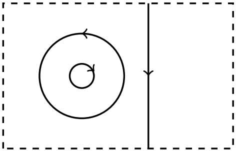


- Consequence



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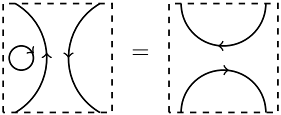

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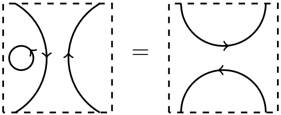

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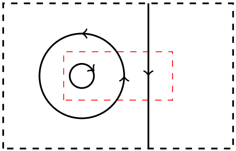

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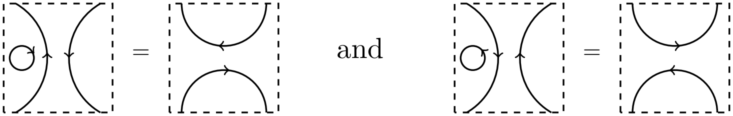

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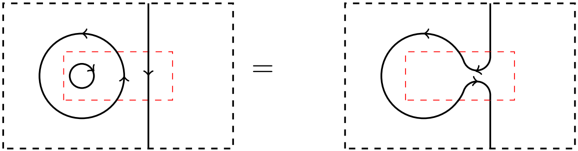

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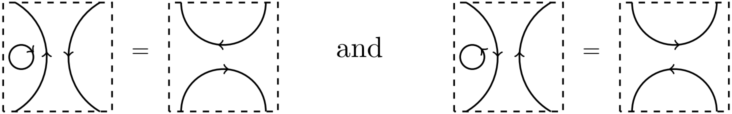

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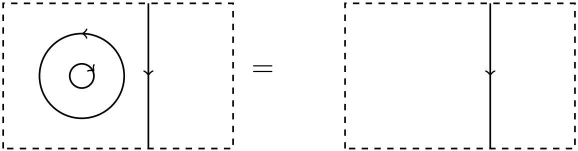

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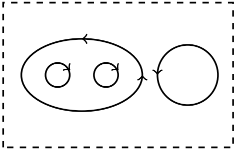

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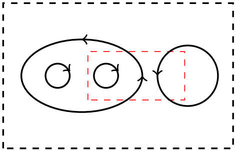

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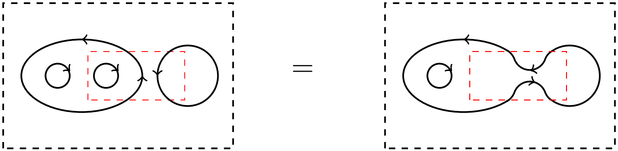

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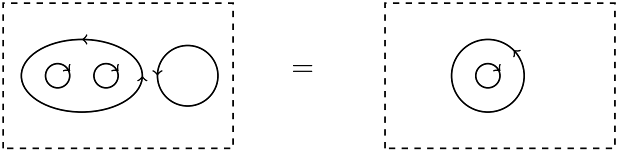

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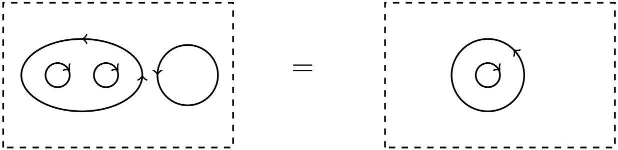

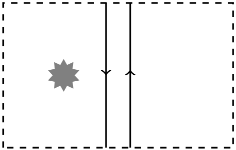

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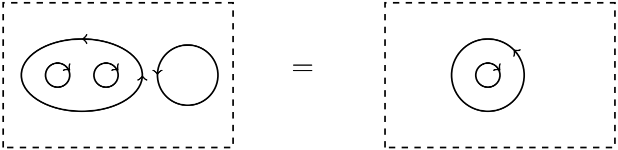

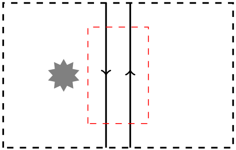

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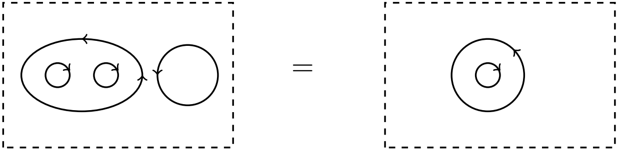

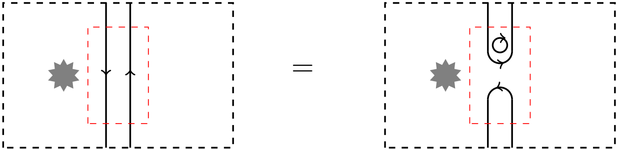

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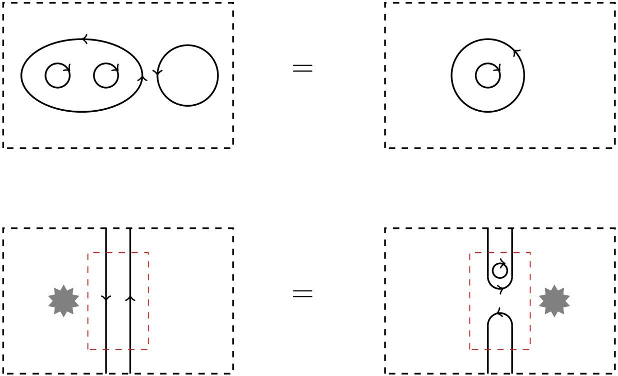

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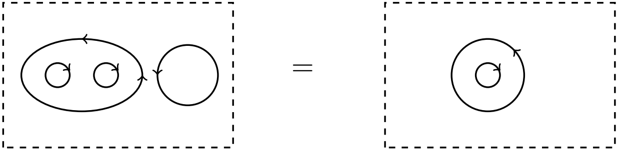

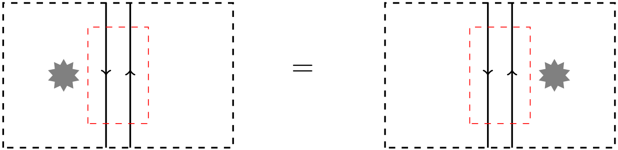

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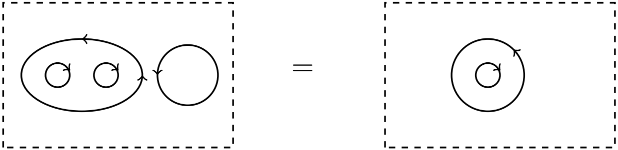

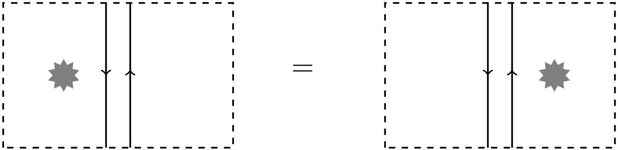

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- Notation

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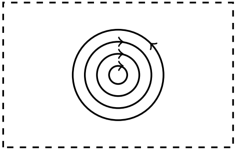

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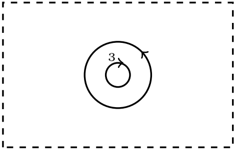

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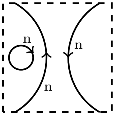

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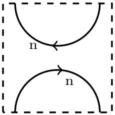

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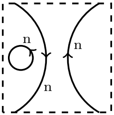

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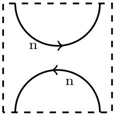

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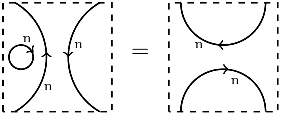

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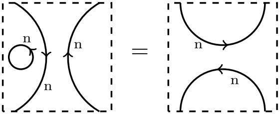

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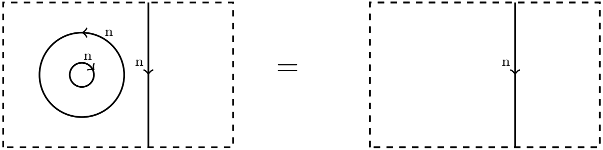

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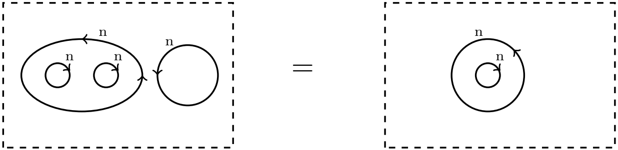

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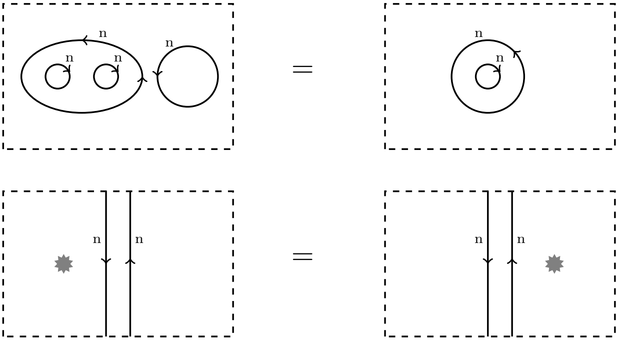

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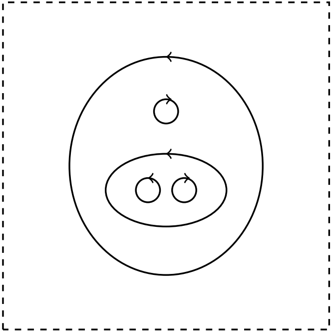

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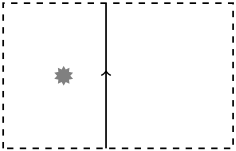

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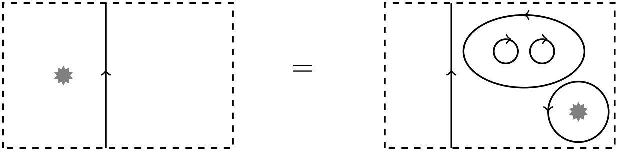

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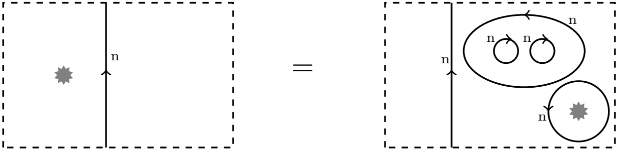

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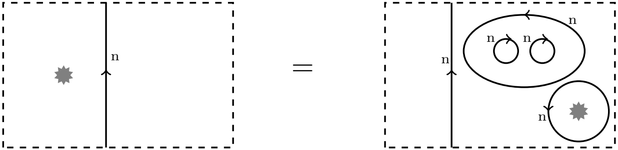

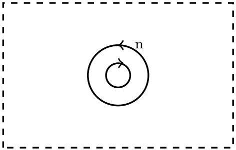

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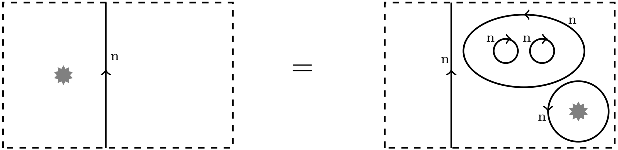

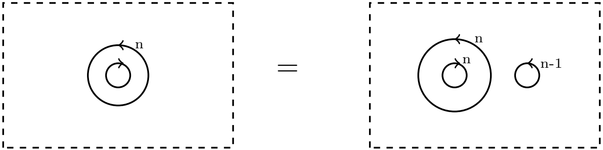

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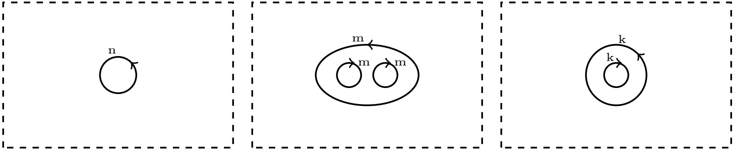

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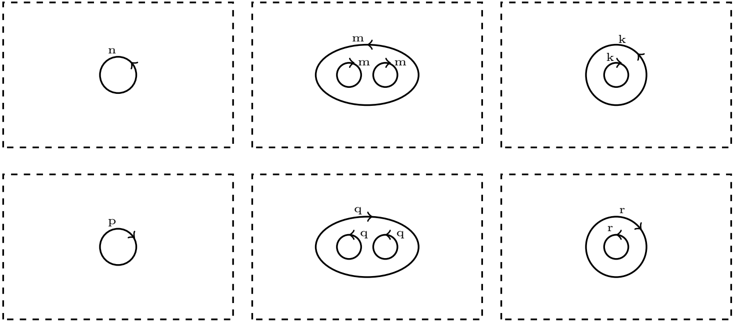

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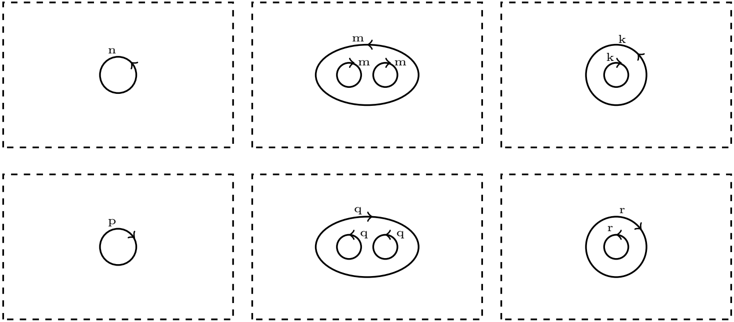

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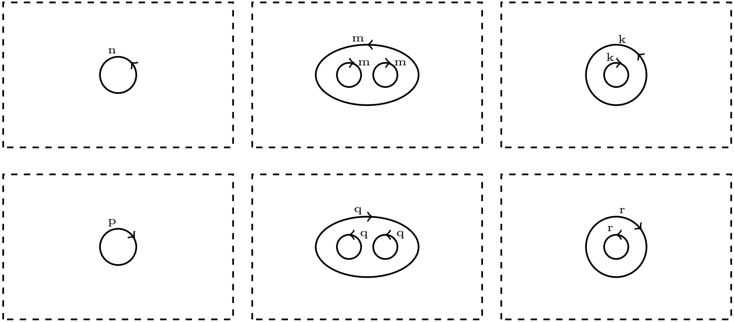

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- For example $n \neq m$ and there is at most one k with $k > n$ and $k > m$.

Example

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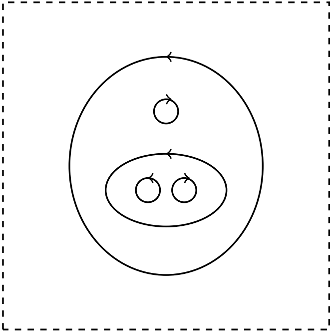

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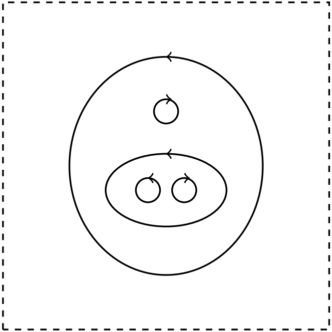

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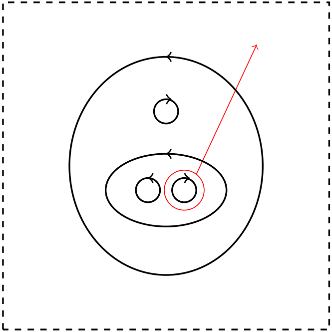

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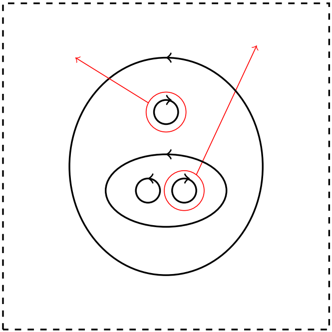

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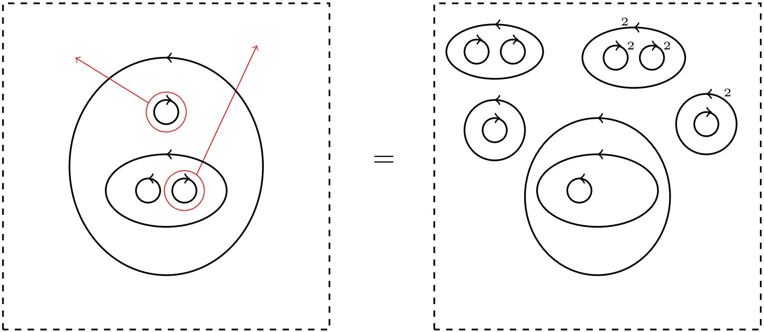

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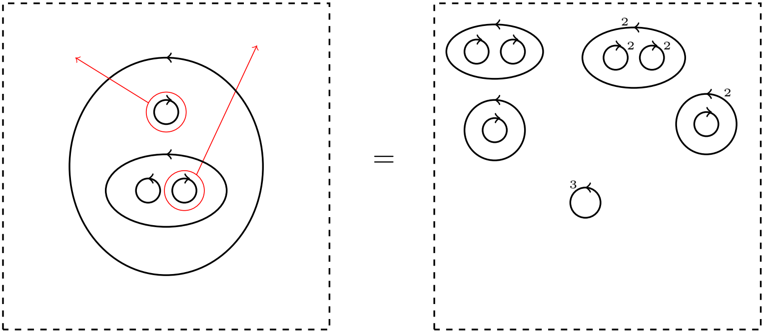

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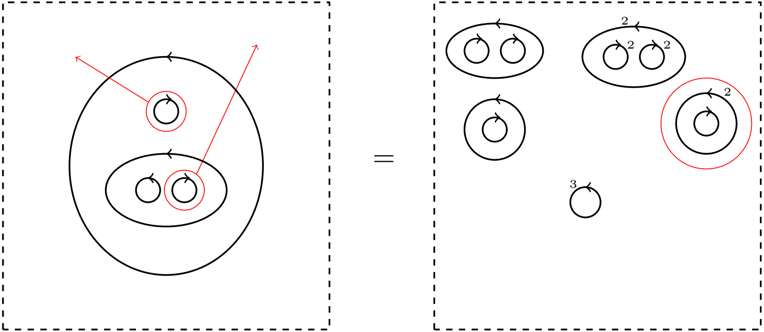

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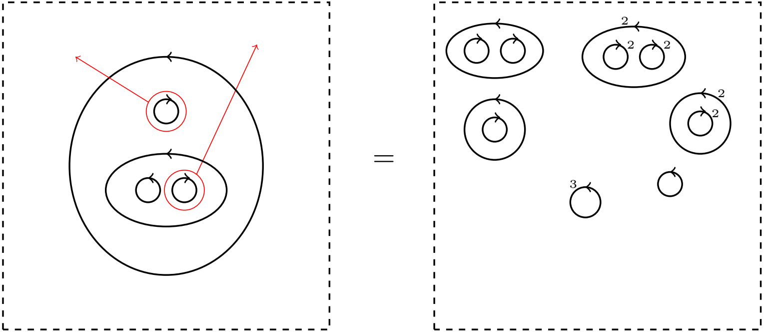

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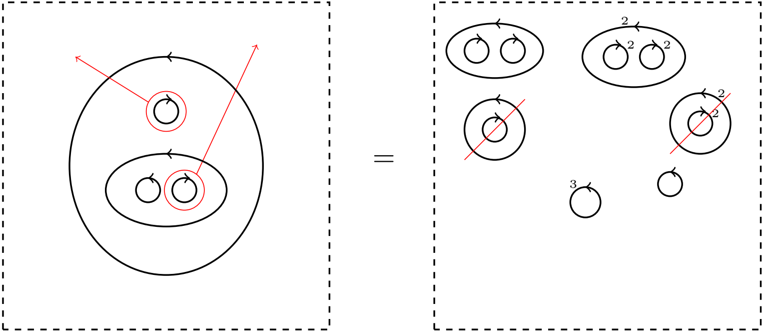

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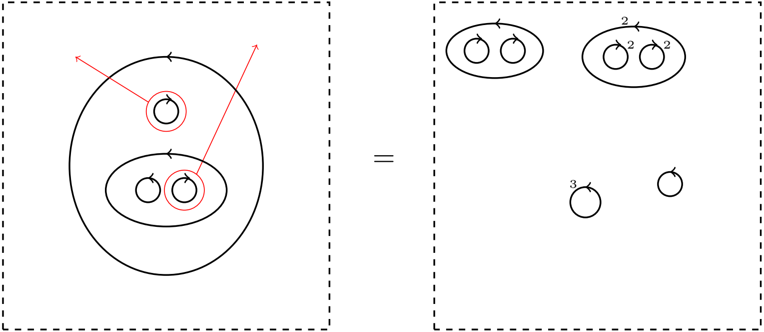

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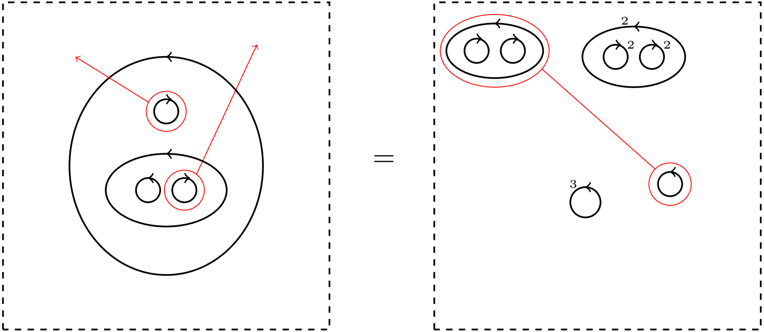

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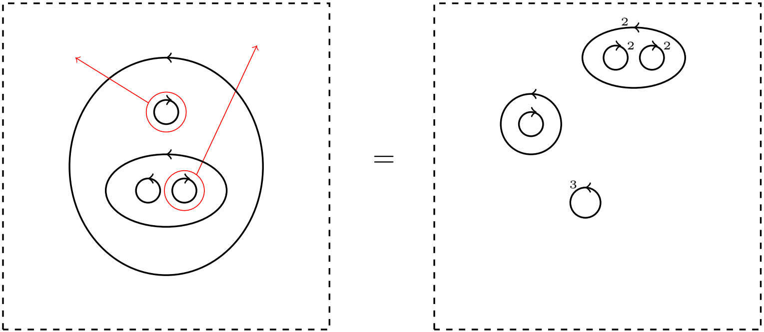

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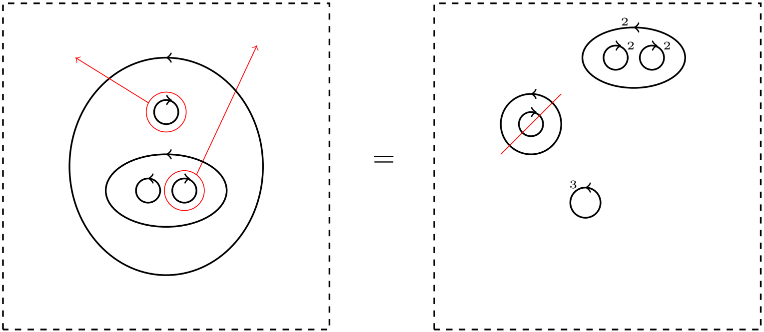

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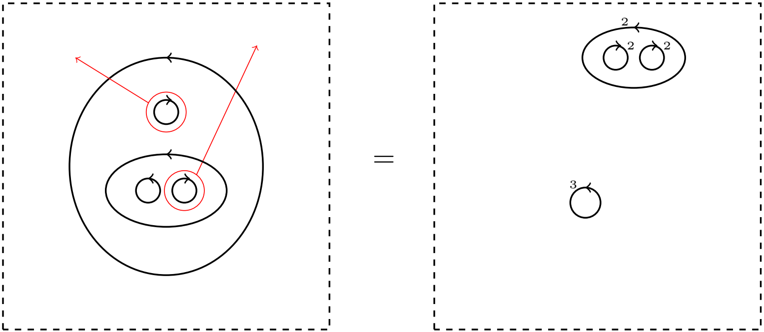

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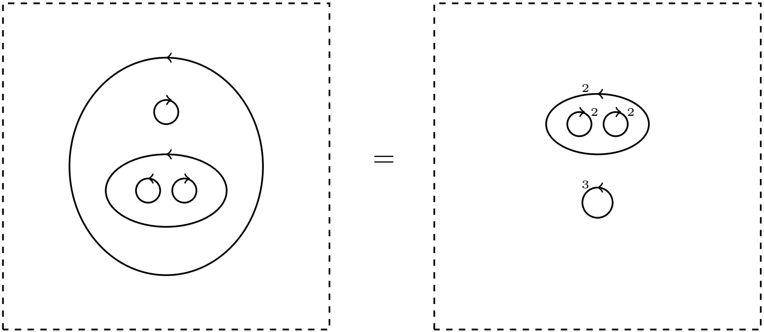

Sequences of Integers

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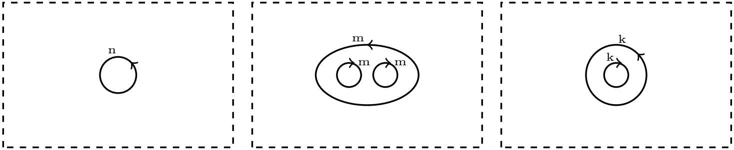

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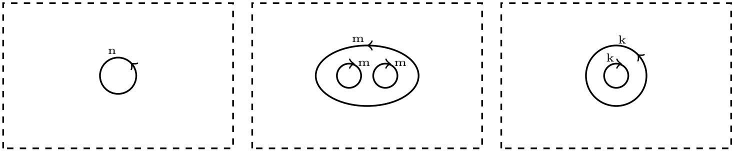

$(\overset{\star}{-1}, 0, \cdots, 0, 1)$ $(\overset{\star}{1}, 0, \cdots, 0, -1)$ $(\overset{\star}{0}, 0, \cdots, 0, 0)$

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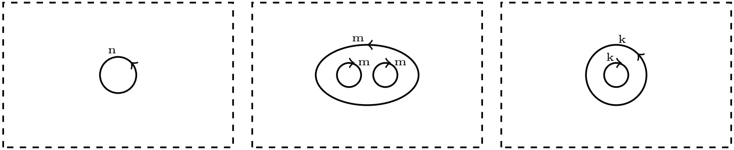
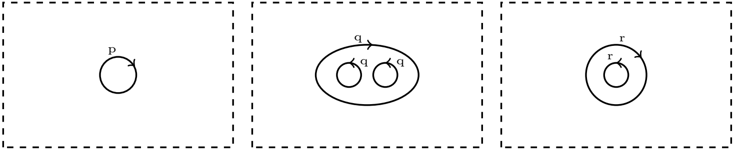


$$(\overset{\star}{-1}, 0, \cdots, 0, 1) \qquad (\overset{\star}{1}, 0, \cdots, 0, -1) \qquad (\overset{\star}{0}, 0, \cdots, 0, 0)$$



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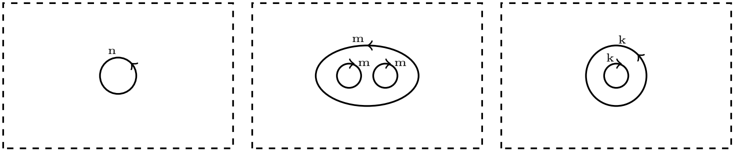
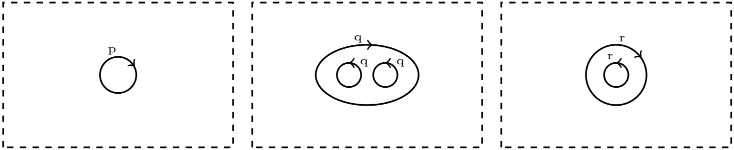


$(\overset{\star}{-1}, 0, \cdots, 0, 1)$ $\quad (\overset{\star}{1}, 0, \cdots, 0, -1)$ $\quad (\overset{\star}{0}, 0, \cdots, 0, 0)$



$(1, 0, \cdots, 0, \overset{\star}{-1})$ $\quad (-1, 0, \cdots, 0, \overset{\star}{1})$ $\quad (0, 0, \cdots, 0, \overset{\star}{0})$

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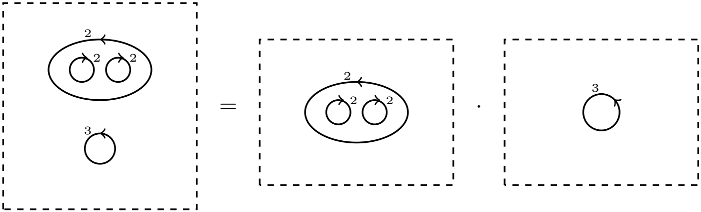

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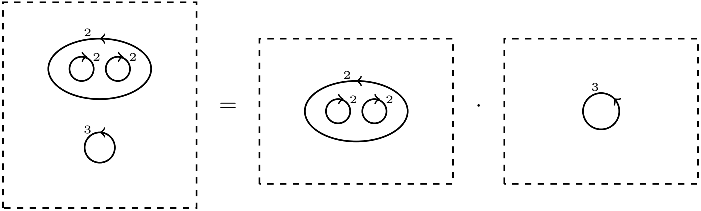

$$(\overset{\star}{0}, 0, -1, 1) = (\overset{\star}{1}, 0, -1) + (\overset{\star}{-1}, 0, 0, 1)$$

Current Work

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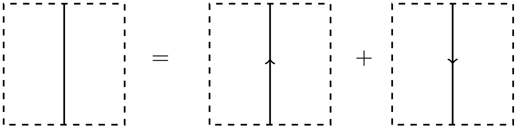
- The $\mathcal{TL}$ planar algebra can be thought of as sitting inside the $\mathcal{DTL}$ planar algebra if we define



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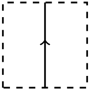
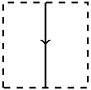
$p_1 \cong$  $\oplus$ 

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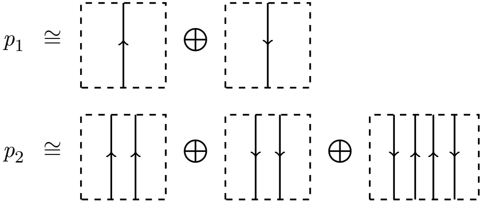

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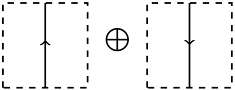
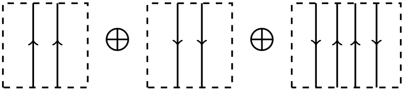
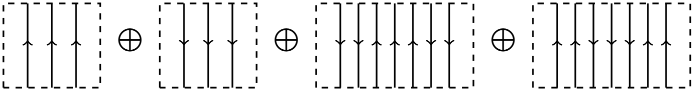
$p_1 \cong$ 

$p_2 \cong$ 

Is $p_3 \cong$  ?

Current Work

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Conjecture

p_n is isomorphic to a direct sum of $n+1$ diagrams, each consisting of vertical strands with a sequence of up or down orientations.

Thank you!