Where do the Planes intersect?

a) $2x + y - z = 2$
$3x - 2y + z = -1$

Since the normals $n_1 = (2,1,-1)$ and $n_2 = (3,-2,1)$ do not satisfy $n_1 = r n_2$ for any $r \in \mathbb{R}$,

They intersect in a line.

Any line in a plane will be perpendicular the the normal of that plane,

So the line where the 2 planes intersect will be perpendicular to both normals.

So compute $n_1 \times n_2$ to get the direction of the line.

$$\begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 3 & -2 & 1 \end{vmatrix} = i(1-2) - j(2+3) + k(-4-3) = -i - 5j - 7k$$

Thus the line is $(-1,-5,-7)t + a$ where a is any point on the line.

So choose a point on both planes

Could pick $x = 0$ to simplify the equations to be

$$\begin{array}{rl} & y - z = 2 \\ + & -2y + z = -1 \\ \hline & -y = 1 \end{array}$$

So $y = -1$ and then $z = -3$

The point is then $(0,-1,-3) = a$

So the line is $(-1,-5,-7)t + (0,-1,-3)$

Where do the Planes intersect?

b) $2x + y - z = 2$ $\quad n_1 = (2, 1, -1)$

$-4x - 2y + 2z = -4$ $\quad n_2 = (-4, -2, 2)$

Since $n_2 = -2n_1$, these are the <u>same</u> plane <u>OR</u> <u>parallel</u> planes

If they intersect at all, then the same plane. Otherwise parallel.

If we find a point on one and not the other, then they are not the same and thus parallel.

So note $(0, 2, 0)$ is on the 1^{st} plane and the second plane.

Thus they are the same plane