How to convert a plane written parametrically $p = a + tv + sw$ to the form $Ax + By + Cz = D$

Example: $p = (1,2,3) + t(0,1,2) + s(1,0,1)$

Find the normal vector to the plane

$$\begin{vmatrix} i & j & k \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{vmatrix} = i + 2j - k \qquad \text{Now we know } A, B, C$$

So the plane is $x + 2y - z = D$

To find D, plug in any pt on the plane, the easiest is $(1,2,3)$:

$$1 + 4 - 3 = D$$
$$2 = D$$

So the plane is $x + 2y - z = 2$

In general: $p = a + tv + sw$

normal vector $v \times w$: $\begin{vmatrix} i & j & k \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} = i(v_2 w_3 - v_3 w_2) - j(v_1 w_3 - v_3 w_1) + k(v_1 w_2 - v_2 w_1)$

So $A = v_2 w_3 - v_3 w_2$

$B = -v_1 w_3 + v_3 w_1$

$C = v_1 w_2 - v_2 w_1$

while

$D = Aa_1 + Ba_2 + Ca_3$

Then the plane is $Ax + By + Cz = D$.

Convert the plane $3x+2y-z=1$ to the form

~~[illegible]~~ $a + tu + sw$ (parametric)

u, w : 2 vectors normal to $(3,2,-1)$ and ~~[illegible]~~ not on same line

a : any pt on the plane

$$(3,2,-1) \cdot (u_1, u_2, u_3) = 0$$

when $3u_1 + 2u_2 - u_3 = 0$ OR $u_3 = 3u_1 + 2u_2$

Choose $u_1 = 0,\ u_2 = 1$ get $(0,1,2) = u$

" $u_1 = 1,\ u_2 = 0$ get $(1,0,3) = w$

Check ~~[illegible]~~

$(0,1,2) \neq r(1,0,3)$ so not on same line

(a_1, a_2, a_3) is on the plane if $3a_1 + 2a_2 - a_3 = 1$

OR $a_3 = 3a_1 + 2a_2 - 1$

Choose $a_1 = 0,\ a_2 = 0$ then $a_3 = -1$

$a = (0,0,-1)$

So plane is $p = (0,0,-1) + t(0,1,2) + s(1,0,3)$