Math 5B - Weekly quiz III

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Find the length of the curve $c(t) = (e^t \cos t, e^t \sin t)$, $0 \le t \le \pi$.

$$\ell(c) = \int_0^{\pi} \| c'(t) \| \, dt = \int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$$

$$\frac{dx}{dt} = e^t \cos t - e^t \sin t \qquad \frac{dy}{dt} = e^t \sin t + e^t \cos t$$

$$\left(\frac{dx}{dt}\right)^2 = e^{2t}\cos^2 t - 2e^{2t} \cos t \sin t + e^{2t} \sin^2 t$$

$$= e^{2t} - 2e^{2t} \cos t \sin t$$

$$\left(\frac{dy}{dt}\right)^2 = e^{2t} \sin^2 t + 2e^{2t} \cos t \sin t + e^{2t} \cos^2 t$$

$$= e^{2t} + 2e^{2t} \cos t \sin t$$

$$\ell(c) = \int_0^{\pi} \sqrt{2e^{2t}} \, dt = \sqrt{2} \int_0^{\pi} e^t \, dt = \sqrt{2} \left(e^t \Big|_0^{\pi} \right)$$

$$\boxed{= \sqrt{2} \left(e^{\pi} - 1 \right)}$$