Version B #1 , Version D #5

$f(x,y) = x^2y^2 \qquad g(x,y) = x^2+4y^2 = 24$

$\nabla f = \lambda \nabla g$: $\quad f_x = 2xy^2 = \lambda \cdot 2x$

$\qquad\qquad\qquad f_y = 2x^2y = \lambda \cdot 8y$

$$\frac{2xy^2}{2x} = \frac{2x^2y}{8y}$$ OR $x=0$ or $y=0$

$$y^2 = \frac{x^2}{4}$$

$$x = \pm 2y$$

Plug in to constraint

$x=0$: $\quad 4y^2 = 24$

$\qquad y = \pm\sqrt{6}$

$y=0$: $\quad x^2 = 24$

$\qquad x = \pm\sqrt{24}$

Plug into constraint

$$4y^2+4y^2=24$$

$$y^2 = 3$$

$$y = \pm\sqrt{3}$$

$$x = \pm 2\sqrt{3}$$

So, consider the points

$(0, \pm\sqrt{6})$, $(\pm\sqrt{24}, 0)$, and $(\pm 2\sqrt{3}, \pm\sqrt{3})$

Plug in to f

$f(0, \pm\sqrt{6}) = f(\pm\sqrt{24}, 0) = 0$ all 4 pts min's

$f(\pm 2\sqrt{3}, \pm\sqrt{3}) = 36$ all 4 pts max's

Version B # 2 , Version D # 4

$a(t) = i + e^t j + 2t k$

$v(t) = (t + c_1,\ e^t + c_2,\ t^2 + c_3)$

$v(0) = 0 \Rightarrow c_1 = 0 \quad c_2 = -1 \quad c_3 = 0$

$v(t) = (t,\ e^t - 1,\ t^2)$

$r(t) = (\frac{1}{2}t^2 + c_1,\ e^t - t + c_2,\ \frac{1}{3}t^3 + c_3)$

$r(0) = i + k \Rightarrow$ $c_1 = 1$

$c_2 = -1$

$c_3 = 1$

$r(t) = (\frac{t^2}{2} + 1,\ e^t - t - 1,\ \frac{1}{3}t^3 + 1)$

Version B # 3 , Version D # 1

$f(x,y,z) = x^2 + 2y^2 + 3z^2 = 1$

$2x = 3\lambda$

$4y = -\lambda$

$6z = 3\lambda$

parallel to plane
$3x - y + 3z = 1$

normal : $(3, -1, 3)$

$\lambda = 2z$ $\quad 2x = 6z$ $\quad 4y = -2z$

$x = 3z$ $\quad y = -\frac{z}{2}$

$(3z, -\frac{z}{2}, z)$ or $(6a, -a, 2a)$

$(6a)^2 + 2(-a)^2 + 3(2a)^2 = 1$

$36a^2 + 2a^2 + 12a^2 = 1$

$a^2 = \frac{1}{50}$

$a = \pm\sqrt{\frac{1}{50}}$

pts : $\left(\frac{6}{\sqrt{50}}, \frac{-1}{\sqrt{50}}, \frac{2}{\sqrt{50}}\right)$ and $\left(\frac{-6}{\sqrt{50}}, \frac{1}{\sqrt{50}}, \frac{-2}{\sqrt{50}}\right)$

$\frac{6}{5\sqrt{2}}$

$\frac{6\sqrt{2}}{5\cdot 2}$

$\frac{3\sqrt{2}}{5}$

Version B # 4, Version D # 3

pt: $(-1, 2, 1)$

line: intersection of planes

$$x + y - z = 2 \quad \text{and} \quad 2x - y + 3z = 1$$

Find plane containing pt & line.

$$\begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 2 & -1 & 3 \end{vmatrix} = (2, -5, -3) \quad \text{vector on plane}$$

Another vector: Find a pt on line

$$\begin{aligned} x + y - z &= 2 \\ 2x - y + 3z &= 1 \end{aligned} \qquad \text{if } z = 0, \quad \begin{aligned} x + y &= 2 \\ + \quad 2x - y &= 1 \\ \hline 3x &= 3 \\ x &= 1 \\ y &= 1 \end{aligned}$$

a pt $(1, 1, 0)$

given pt

vector: $(-1, 2, 1) - (1, 1, 0)$
$= (-2, 1, 1)$

normal to plane: $\begin{vmatrix} i & j & k \\ 2 & -5 & -3 \\ -2 & 1 & 1 \end{vmatrix} = (-2, 4, -8)$

$-2x + 4y - 8z = d$

(Plug in $(1, 1, 0)$)

$-2 + 4 = 2 = d$

$\therefore$ Plane is given by: $\boxed{-2x + 4y - 8z = 2}$

Version B # 5, Version D # 2

$f(x,y) = x^3 + y^3 - 3xy + 4$

$f_x = 3x^2 - 3y \quad = 0 \quad$ when $\quad x^2 = y$

$f_y = 3y^2 - 3x \quad = 0 \quad$ when $\quad x = y^2$

So both when $x = x^4$, $x = 1$ or $x = 0$

So $y = 1$ and $x = 1$ or $y = 0$ and $x = 0$

$(1,1)$ or $(0,0)$

$f_{xx} = 6x$

$f_{xy} = -3$

$f_{yy} = 6y$

$D(x,y) = f_{xx} f_{yy} - f_{xy}^2 = 36xy - 9$

$D(0,0) = -9 < 0$ saddle

$D(1,1) = 27 > 0$ max or min

$f_{xx}(1,1) = 6 > 0$ min