Version C # 1, Version A # 3

$f(x,y,z) = x^2 - y^2 + 2z^2 = 1$

$f_x = 2x = 2\lambda$

$f_y = -2y = 4\lambda$

$f_z = 4z = 6\lambda$

parallel to line joining $(3,-1,0)$ and $(5,3,6)$

This line is in direction $(2,4,6)$

so get

pt $(x, -2x, \frac{3}{2}x)$

OR $(2a, -4a, 3a)$

$x^2 - y^2 + 2z^2 = 1$

$(2a)^2 - (-4a)^2 + 2(3a)^2 = 1$

$4a^2 - 16a^2 + 18a^2 = 1$

$6a^2 = 1 \qquad a = \pm\sqrt{\frac{1}{6}}$

pt : $\left(\frac{2}{\sqrt{6}}, \frac{-4}{\sqrt{6}}, \frac{3}{\sqrt{6}}\right)$ and

$\left(-\frac{2}{\sqrt{6}}, \frac{4}{\sqrt{6}}, \frac{-3}{\sqrt{6}}\right)$

Version C #2 , Version A #5

$a(t) = (t, t^2, \cos 2t)$

$v(t) = (\frac{1}{2}t^2 + C_1, \frac{1}{3}t^3 + C_2, \frac{1}{2}\sin 2t + C_3)$

$v(0) = (1, 0, 1) \Rightarrow C_1 = 1, C_2 = 0, C_3 = 1$

$v(t) = (\frac{1}{2}t^2 + 1, \frac{1}{3}t^3, \frac{1}{2}\sin 2t + 1)$

$r(t) = (\frac{1}{6}t^3 + t + C_1, \frac{1}{12}t^4 + C_2, -\frac{1}{4}\cos 2t + t + C_3)$

$r(0) = (0, 1, 0) \Rightarrow C_1 = 0, C_2 = 1, C_3 = \frac{1}{4}$

$r(t) = (\frac{1}{6}t^3 + t, \frac{1}{12}t^4 + 1, -\frac{1}{4}\cos 2t + t + \frac{1}{4})$

Version C #3 Version A #4

$f(x,y) = xe^y$ $g(x,y) = x^2 + y^2 = 2$

$$\left.\begin{array}{l} f_x = \lambda g_x \\ f_y = \lambda g_y \end{array}\right\} \quad \begin{array}{l} e^y = \lambda \cdot 2x \\ xe^y = \lambda \cdot 2y \end{array}$$

$$\frac{e^y}{2x} = \frac{xe^y}{2y}$$

OR $x=0$ & $y=0$

But $(0,0)$ is not on $g=2$, so not considered

$$2ye^y = 2x^2 e^y$$

$$y = x^2$$

On $g = 2$?

$$x^2 + x^4 = 2$$

$$\underbrace{(x^2+2)}_{\text{never } 0}(x^2-1) = 0$$

$x = \pm 1 \Rightarrow y = 1$

Check: $f(1,1) = e$ ← max

$f(-1,1) = -e$ ← min

Version C #4   Version A #1

$f(x,y) = x^4 + y^4 - 4xy + 2$

$f_x = 4x^3 - 4y = 0$ when $x^3 = y$

$f_y = 4y^3 - 4x = 0$ when $y^3 = x$

So both when $y^9 = y$ $\Rightarrow$ $y = \pm 1$, $x = \pm 1$
OR $x = 0 = y$

Crit pts $(-1,-1)$, $(1,1)$, and $(0,0)$

$f_{xx} = 12x^2$
$f_{yy} = 12y^2$
$f_{xy} = -4$

$D(x,y) = 144x^2y^2 - 16$

$D(\pm 1, \pm 1) = 144 - 16 > 0$
either max or min

$f_{xx}(\pm 1, \pm 1) = 12 > 0$ min

$D(0,0) = -16 < 0$ saddle

So $(1,1)$ and $(-1,-1)$ are both mins

and $(0,0)$ is a saddle

Version C # 5 , Version A #2

pt $(1,2,3)$

line $x = 4-2t$ $y = 3+5t$ $z = 7+4t$

Find plane containing pt & line

one vector on the plane is that of the direction of the line

$(-2, 5, 4)$

Need another vector:

$(1,2,3) - (4,3,7)$

$= (-3,-1,-4)$ OR choose $(3,1,4)$

normal to plane:

$$\begin{vmatrix} i & j & k \\ -2 & 5 & 4 \\ 3 & 1 & 4 \end{vmatrix} = (16, 20, -17)$$

$16x + 20y - 17z = d$

$16 + 20(2) - 17(3) = d$

$5 = d$

$$\boxed{16x + 20y - 17z = 5}$$