NAME:_____

SECTION (circle one) : 8am 5pm 6pm 7pm

Math 3C Quiz 1, January 11, 2011

Find the general solution for the following differential equation:

$$y' = y(1-y)$$

$$\frac{dy}{dt} = y(1-y)$$

$$\int \frac{1}{y(1-y)}\, dy = \int dt$$

$$\frac{1}{y(1-y)} = \frac{A}{y} + \frac{B}{1-y}$$

$$\int \frac{1}{y} + \frac{1}{1-y}\, dy = t + C$$

$$\ln|y| - \ln|1-y| = t + C$$

$$\frac{y}{1-y} = Ce^t \quad \text{or}$$

$$\frac{y-1+1}{1-y} = Ce^t$$

$$-1 + \frac{1}{1-y} = Ce^t$$

$$\frac{1}{Ce^t+1} = 1-y$$

$$y = 1 - \frac{1}{Ce^t+1}$$

$$y = \frac{Ce^t}{1+Ce^t}$$

NAME:_____

SECTION (circle one) : 8am 5pm 6pm 7pm

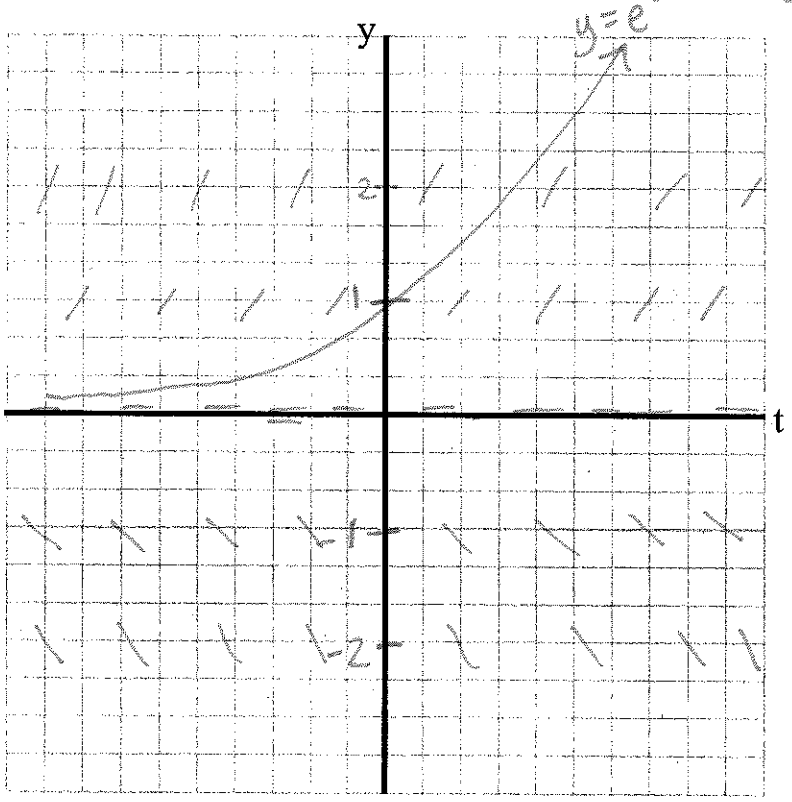
Math 3C Quiz 2, January 18, 2011

Draw the slope field for the following differential equation: $y' = y$. Then solve the following initial value problem and plot this solution on the slope field.

$$y' = y, \quad y(0) = 1$$

equil. soln?
$0 = y$ ✓

isoclines?
$c = y$
horizontal lines



$\frac{dy}{dt} = y$

$\int \frac{dy}{y} = \int dt$

$\ln|y| = t + C$

$y = Ce^t$

$y(0) = C = 1$

$y = e^t$

NAME:_____

SECTION (circle one) : 8am 5pm 6pm 7pm

Math 3C Quiz 3, January 25, 2011

Solve the following initial value problem.

$$ty' + y = 2t, \quad y(1) = 3$$

$y' + \frac{1}{t} y = 2$

$\mu = e^{\int \frac{1}{t} dt} = e^{\ln t} = t$

$t\left(y' + \frac{1}{t} y\right) = 2t$

$\frac{d}{dy}(yt) = 2t$

$yt = \int 2t \, dt$

$yt = t^2 + C$

$y = t + \frac{C}{t}$

$y(1) = 1 + C = 3 \Rightarrow C = 2$

$$\boxed{y = t + \frac{2}{t}}$$

NAME:_____

SECTION (circle one) : 8am 5pm 6pm 7pm

Math 3C Quiz 4, February 1, 2011

A tank initially contains 100 gallons of fresh water, but then a salt solution of unknown concentration x lb/gal is poured into the tank at 2 gal/min. The well-stirred mixture flows out of the tank at the same rate. After 30 minutes, the concentration of salt in the tank is 2.4 lb/gal. What is the concentration (in lb/gal) of the entering brine?

$y(t)$ = lbs of salt @ time t (mins.)

$y(0) = 0$ since initially fresh water

$$y' = x \frac{lb}{gal} \cdot 2 \frac{gal}{min} - \frac{y\ lb}{100\ gal} \cdot 2 \frac{gal}{min}$$

$$y' = 2x - \frac{y}{50}$$

$$y' + \frac{1}{50} y = 2x$$

$$\mu = e^{\int \frac{1}{50} dt} = e^{t/50}$$

$$\mu \left(y' + \frac{y}{50}\right) = \mu(2x)$$

$$\frac{d}{dy}(\mu y) = 2x e^{t/50}$$

$$y e^{t/50} = \int 2x e^{t/50} dt$$

$$= 100x e^{t/50} + C$$

So $y = 100x + C e^{-t/50}$

Use $y(0) = 0$ to find C:

$$y(0) = 100x + C = 0$$

$$\Rightarrow C = -100x$$

Use that $y(30) = (2.4)(100)$ lbs $= 240$ lbs

to find x:

$$y = 100x\left(1 - e^{-t/50}\right)$$

$$y(30) = 100x\left(1 - e^{-3/5}\right) = 240$$

$$\boxed{x = \frac{2.4}{1 - e^{-3/5}} \text{ lbs/gal}}$$