MATH 34A REVIEW FOR MIDTERM 1, WINTER 2012

WARNING: This is a mesh of problems from the various times I have taught 34A. This is supplemental material for your studying pleasure. Please consult the syllabus and your lecture notes (available on GauchoSpace) to see exactly what sections of the book may be on the test.

1. WORD PROBLEMS

There are a lot of word problems in chapter 3. If you want more word problems to study from, consult chapter 11. The practice midterms at the end of the book are good practice as well.

When you are presented with a word problem you should clearly identify the unknowns. Then interpret the information provided in the problem as equations involving your unknowns. Lastly, solve the equations and present the answer in terms of whatever is asked. The most important part is to write clearly so that a grader can follow your work.

We'll do some in discussion section.

2. ALGEBRA

(1) Simplify $(x + \frac{1}{x})(x^2 + 1)^{-1} + \frac{1}{x}$.
(2) Solve for x: $\frac{2x+d}{x+1} = a$.
(3) Solve the following system of equations:
$2x + 3y = a$
$x + y = b$.
(4) There are two positive numbers. Four times the small number plus 3 times the big number is 46. Two times the small number plus the big number is 19. What is 10 times the big number minus 6 times the small number?
(5) I have three numbers. The biggest one is twice the middle one, and the biggest one plus the middle one is four times the smallest one. The smallest one plus the middle one is two less than the biggest one. What are the numbers?

3. GEOMETRY

(1) A circular can has height h and the base is a circle with radius R. If the volume is 4π express R in terms of h.
(2) The perimeter of a rectangle equals $3/2$ times its area. Express the length of the rectangle in terms of the width.
(3) A window has the shape of a semi-circle placed on top of a square. The glass costs $7 per square meter. Write the total cost of the window in terms of the radius of the circle.

(4) What is the distance between the points (-3,8) and (5,2)? Draw a picture.
(5) What is the length of the hypotenuse of a right angled triangle when the other two sides have lengths 5 and 12?
(6) The vertical mast of a yacht is 40 feet high. A rope runs in a straight line from the top to a pulley 30 feet horizontally from the base of the mast. How many feet long is the rope?

4. Units

(1) If the units of A is dollars/meters and the units of B is meters. What are the units of A/B and AB?
(2) A swimming pool is 3 meters deep, 5 meters wide, and 80 meters long. Water is pumped into the pool at a rate of 100 liters per minute. How many hours will it take to fill the pool? (Remember that there are 1000 liters in a cubic meter).
(3) If a sphere has radius 7 inches, what is the volume of the sphere in pints? The volume of a sphere is $\frac{4}{3}\pi r^3$. Also use that 1 foot = 12 inches, 1 ft^3 = 7.5 gallons, and 1 gallon = 8 pints. You do not need to multiply out.
(4) If the radius of the sphere is tripled, the volume is how many times as big?

5. Percentages

(1) A manager starts with a salary of 50,000 dollars. After one year he received a 25% pay rise. After another year his pay is cut by 10%. What is his salary after this?
(2) If 1000 liters of water with an unknown chlorine content are combined with 500 liters of water with 45 ppm (parts per million) of chlorine the result is 75 ppm chlorine. What is the concentration of chlorine in the unknown water?
(3) There are 3 beakers each of which contains saline solution. Beaker A initially contains 3 liters of 10% salt solution. Beaker B initially contains 2 liters of 20% salt solution. Beaker C initially contains 4 liters of 0% salt solution. Two liters are transferred from A to B and the result is thoroughly mixed. Then one liter is transferred from B to C and the result mixed. Finally two liters are transferred from C back to A. What is the percentage concetration of salt in A after all this?
(4) I have milk that contains 1 percent fat and milk that contains 4 percent fat. A customer wants a double latte made with 1/2 of a pint of 2 percent milk. How much of each type of milk should I use?
(5) If I combine x liters of blue paint with y liters of red paint what percentage of blue paint is in the combination?

6. Functions

(1) What is the inverse of the function $f(x) = \frac{a}{x}$?
(2) What is the inverse of the function $f(x) = 2x - 1$?
(3) What is the inverse of the function $f(x) = \frac{2x+1}{3x-5}$?
(4) What is the inverse of the function $f(x) = 8x^3$?

(5) The function $f(x)$ converts degrees Celsius to degrees Fahrenheit. What does the inverse function do? Recall that $f(x)$ is a line. Further recall that water freezes at 0 degrees Celsius or 32 degrees Fahrenheit and boils at 100 degrees Celsius or 212 degrees Fahrenheit. Derive a formula for $f(x)$. What is $f(30)$?

7. CAR PROBLEMS

(1) UCSB and LAX are 120 miles apart. The Santa Barbara airbus leaves LAX at 3pm and drives to UCSB at an average speed of 30 mph. You leave UCSB at 4pm driving at 90 mph on the same road towards LAX. What time do you pass the airbus?

(2) Car A leaves SB at noon driving north at 49 mph. Car B leaves at 1 pm traveling the same route at a different constant speed. How fast should Car B go in order to catch up with Car A by 4:30pm?

(3) A highway patrolman traveling at the speed limit is passed by a car going 15 mph faster than the speed limit. After one minute, the patrolman speeds up to 90 mph. How long after speeding up until the patrolman catches up with the speeding car. The speed limit is 65 mph.

(4) Suppose Car A's distance in miles north of SB t hours after noon is given by the function $A(t) = 300 - 90t$ and Car B's distance in miles north of SB t hours after noon is given by the function $B(t) = 60t$. Describe the scenario. Calculate when and where the cars meet.

8. ERROR AND LIMITS

(1) If you think a carton of milk is 1/5 full but it is really 1/4 full, what is the percentage error?

(2) You have \$55 in your pocket, but you think you have \$60. What is the percentage error?

(3) Calculate the following limit: $\lim\limits_{n\to\infty} \dfrac{5}{n^3}$.

(4) Calculate the following limit: $\lim\limits_{x\to 3} \dfrac{x^3 - 2x - 3}{x - 3}$.

(5) Calculate the following limit: $\lim\limits_{x\to\infty} (x^{-2} + 2)$.

9. CHANGE OF A FUNCTION AND SUMMATION

(1) If x is increased from 6 to 7 how much does $\frac{1+x}{2+x}$ change by? Does the function increase or decrease when x goes from 6 to 7?

(2) (a) Find
$$\frac{\text{the change in } (x-2)(x+2) \text{ as } x \text{ increases from 1 to } 1+h}{h}$$
where h is a positive real number. Simplify your answer.

(b) Find

$$\lim_{h \to 0} \frac{\text{the change in } (x-2)(x+2) \text{ as } x \text{ increases from } 1 \text{ to } 1+h}{h}$$

(3) Calculate $\displaystyle\sum_{i=0}^{n} i^3$

(4) Calculate $\displaystyle\sum_{n=1}^{4} (n-1)$