Math 118: Homework 2

1. (a) Give an example of a function $f : X \to Y$ and two subsets A and B of X such that $f(A \cap B) \neq f(A) \cap f(B)$.

(b) Prove that $f : X \to Y$ is one-to-one if and only if $f(A \cap B) = f(A) \cap f(B)$ for all $A \subseteq X$ and $B \subseteq X$.

2. Let $f : X \to Y$ and $g : Y \to Z$. If $A \subseteq Z$, show that $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$. (Recall that the composition $g \circ f : X \to Z$ is defined as, for every $x \in X$, $(g \circ f)(x) = g(f(x))$.)

3. Chapter 2, Exercise #2

4. Chapter 2, Exercise #11

5. Let (X_1, d_1) and (X_2, d_2) be metric spaces. Let $X = X_1 \times X_2$. (Recall this means that $X = \{(x, y) : x \in X_1 \text{ and } y \in X_2\}$.)

(a) Show that the function defined by

$$D_1(v, w) = d_1(x_1, x_2) + d_2(y_1, y_2) \text{ for each } v = (x_1, y_1) \in X \text{ and } w = (x_2, y_2) \in X$$

is a distance on X.

(b) Show that the function defined by

$$D_2(v, w) = [(d_1(x_1, x_2))^2 + (d_2(y_1, y_2))^2]^{\frac{1}{2}} \text{ for each } v = (x_1, y_1) \in X \text{ and } w = (x_2, y_2) \in X$$

is a distance on X.

6. Chapter 2, Exercise #5