Math 118: Homework 5

1. Prove, using the definition of convergence, that the sequence $\left\{ \dfrac{n+1}{n\sqrt{n}} \right\}$ converges.

2. Chapter 3, #2

3. Let $\{p_k\}$ and $\{q_k\}$ be sequences in a metric space X. Define $\{t_n\}$ to be the sequence $\{p_1, q_1, p_2, q_2, p_3, q_3, ..., p_k, q_k, ...\}$. Prove that $\{t_n\}$ converges if and only if $\{p_k\}$ and $\{q_k\}$ both converge and have the same limit.

4. Prove or give a counterexample (consider all the sequences to be in $\mathbb{C}$):

(i) If $\{x_n\}$ converges and $x_n \neq 0$ for all natural numbers n, then $\{\frac{1}{x_n}\}$ converges.

(ii) If $\{x_n\}$ converges to x and $x_n \geq 0$ for all n, then $\{\sqrt{x_n}\}$ converges to $\sqrt{x}$.

(iii) If $\{x_n\}$ and $\{y_n\}$ are two sequences, and the sequence $\{x_n + y_n\}$ converges, then $\{x_n\}$ and $\{y_n\}$ both converge.

(iv) If $\{x_n\}$ and $\{y_n\}$ are sequences and the sequence $\{x_n y_n\}$ converges, then $\{x_n\}$ and $\{y_n\}$ both converge.

5. Chapter 3, #3

6. Chapter 3, #20

7. Chapter 3, #22 (You may use the result of Exercise #21.)