Hint for Section 2.3, #2(a)

Let $u(x,t)$ solve the wave equation (with bounded second derivatives). It might be easier to define the new function v as a function of the variables x and s:

$$(\star) \quad v(x,s) = \frac{c}{\sqrt{4\pi ks}} \int_{-\infty}^{\infty} e^{-t^2c^2/4ks} u(x,t)\, dt.$$

We want to show that $v_s(x,s) = kv_{xx}(x,s)$ – in other words, v solves the heat equation.

Since $u_{tt}(x,t) = c^2 u_{xx}(x,t)$, we know that

$$\frac{c}{\sqrt{4\pi ks}} \int_{-\infty}^{\infty} e^{-t^2c^2/4ks} u_{tt}(x,t)\, dt. = \frac{c}{\sqrt{4\pi ks}} \int_{-\infty}^{\infty} e^{-t^2c^2/4ks} (c^2 u_{xx}(x,t))\, dt.$$

The right-hand side clearly equals $c^2 v_{xx}(x,s)$. Therefore, we only need to show that the left-hand side is $\frac{c^2}{k} v_s(x,s)$ Do this by calculating both

(a) $\dfrac{c^2}{k} v_s(x,s)$ using formula $(\star)$.

(b) the integral on the left-hand side (use integration by parts twice!)

Compare these answers to make sure they're the same!