Homework due Dec. 4th

Math 124A: PDEs (Fall 2007)

- **Problem # 1** Consider the following problem on the half-line:

$$\begin{cases} u_{tt} = 4u_{xx} & \text{on } 0 < x < \infty \\ u(x,0) = \phi(x) \\ u_t(x,0) = 0 \\ u(0,t) = 0 \end{cases}$$

  where $\phi(x) = 1$ for $3 < x < 4$, $\phi(x) = -2$ for $4 < x < 5$. Draw the domain in the $x - t$ plane (for positive times), and write the value of the solution u in each region of the domain where u is constant.

- **Section 3.2 #2** Find and plot the solution of the following problem on the half-line at times $\frac{a}{2c}$, $\frac{a}{c}$, $\frac{2a}{c}$, and $\frac{3a}{c}$.

$$\begin{cases} u_{tt} = c^2 u_{xx} & \text{on } 0 < x < \infty \\ u(x,0) = 0 \\ u_t(x,0) = \psi(x) \\ u_x(0,t) = 0 \end{cases}$$

  where $\psi(x) = V$ (a constant) for $a < x < 2a$ and $\psi(x) = 0$ for $0 < x \leq a$ and $x \geq 2a$.

  Start by drawing a large graph of the domain (for positive times only – ie, draw the first quadrant in the $x - t$ plane). Draw the important characteristics (and their reflections); this should break up the domain into 9 distinct regions. Consider the domain of dependence for points in each region and use d'Alembert's formula to find the solution in each region. If you draw the characteristic lines carefully, it will be a lot easier to find the solution at the specific times given!

- **Section 3.2 #5** Solve $u_{tt} = 4u_{xx}$ on the half-line $0 < x < \infty$ with the conditions $u(x,0) = 1$, $u_t(x,0) = 0$, and $u(0,t) = 0$.

  Use the formulas (2) and (3) in your book to find the solution. Draw the domain. Where is the singularity of the solution located in this domain?

- **Section 3.2 #9** Consider the wave equation $u_{tt} = u_{xx}$ on the finite interval $0 < x < 1$ with initial conditions $u(x,0) = \phi(x) = x^2(1-x)$ and $u_t(x,0) = \psi(x) = (1-x)^2$ on $0 < x < 1$ and with boundary conditions $u(0,t) = u(1,t) = 0$.

  (a) Find $u(\frac{2}{3}, 2)$. Draw the domain with the important characteristics and their reflections. Use d'Alembert's formula to find the solution at the point $\left(\frac{2}{3}, 2\right)$. Remember to consider the domain of dependence for $(\frac{2}{3}, 2)$, and to use the right extension of the initial conditions. (It may help to graph the functions ϕ_{EXT} and ψ_{EXT} and/or write down formulas for them.) The answer in the back of the book for this problem is not right.

  (b) Using the graph of the domain and the correct extension of the initial conditions again, find $u(\frac{1}{4}, \frac{3}{2})$. (Notice that this is a different point than given in the book!)

- **Section 3.4 #2**

- **Section 3.4 #3**