Math 124B: PDEs

Homework #7

Hint for Problem #1 (b):

Use the fact that the derivative of the Heavyside function is the delta function: $H'(x) = \delta(x)$.

Then, remember what the Fourier transform of a derivative is. ($\widehat{H'}(k) = ...?$)

The tricky part is what to do when you divide by 0. Classically, if you have $kF(k) = 0$, all you know about the function F is that $F(k) = 0$ for all $k \neq 0$. Since we are allowing distribution functions, whenever you have $kF(k) = 0$, you can conclude that $F(k) = c\,\delta(k)$ for some constant c.

By this point, you should have found that $H(k) = \dfrac{1}{ik} + c\,\delta(k)$. To figure out the constant, use Fourier's identity! You may assume that

$$\int_{-\infty}^{\infty} \frac{e^{ikx}}{ik}\,dk = \int_{-\infty}^{\infty} \frac{\sin(kx)}{k}\,dk = \begin{cases} \pi & \text{when } x > 0 \\ -\pi & \text{when } x < 0. \end{cases}$$

(The fact that the real integral above converges is easily proved using techniques of complex integration.)