Polar, Cylindrical, and Spherical Coordinates

1. Let $f(x, y) = \dfrac{y(x^2 + y^2)^{\frac{5}{2}}}{x}$

(a) Find $\frac{\partial f}{\partial x}$.

(b) Rewrite f in terms of the polar coordinates r and θ.
(*Hint: Notice that* $f(x, y) = \left(\frac{y}{x}\right)(\sqrt{x^2 + y^2})^5$.)

(c) Find $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial \theta}$. Then use the chain rule, $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial f}{\partial \theta}\frac{\partial \theta}{\partial x}$, to find $\frac{\partial f}{\partial x}$. (Use that $\frac{\partial r}{\partial x} = \cos(\theta)$, $\frac{\partial \theta}{\partial x} = -\frac{\sin(\theta)}{r}$.) (*Can you see why this is the same as the answer that you got in part (a)?*)

2. Recall that spherical coordinates are defined by

$$\begin{aligned} x &= \rho \sin\phi \cos\theta \\ y &= \rho \sin\phi \sin\theta \\ z &= \rho \cos\phi \end{aligned}$$

Show that the Jacobian of this mapping $\frac{\partial(x,y,z)}{\partial(\rho,\phi,\theta)} = \rho^2 \sin\phi$.

3. Find the (x, y, z) point described by the spherical coordinates $(\rho, \phi, \theta) = (4, \frac{\pi}{6}, \frac{\pi}{4})$.

4. Find both the cylindrical coordinates $(r,\ \theta,\ z)$ and the spherical coordinates (ρ, ϕ, θ) that correspond to the point $(x, y, z) = (0,\ 1,\ 1)$. (*Hint: Draw the picture! Check your answers using the equations for $x, y,$ and z in terms of cylindrical coordinates and spherical coordinates.*)