

Finding Limits

Find the value of the limit, if it exists. Otherwise, prove that the limit doesn't exist.

$$(a) \lim_{\substack{x \rightarrow 1 \\ y \rightarrow -1}} \frac{xy}{x + y + 1}$$

$$(b) \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{x^2}{x^2 + y^2}$$

$$(c) \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{(x + 1) \sin y}{y}$$

$$(d) \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} e^{x^2 + y^2}$$

$$(e) \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} e^{-\frac{1}{x^2 + y^2}}$$

$$(f) \text{ Let } f(x, y) = \frac{x}{x-y} \text{ when } x \neq y \text{ and } f(x, x) = 0. \lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} f(x, y)$$