An example of a function where $f_{xy}(0,0) \neq f_{yx}(0,0)$

Consider the continuous function defined piecewise by

$$f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

(a) Find f_x and f_y. (This is easy for $(x,y) \neq (0,0)$. At $(0,0)$, show that f_x and f_y both exist and equal 0 by using the definition of the partial derivative.) The functions f_x and f_y are in fact continuous, and they are also differentiable! (see part (c))

(b) Find $f_{xy}(0,0)$ and $f_{yx}(0,0)$ (Use the definition of partial derivatives again!)

(c) Why doesn't this example contradict the theorem (2.124) in your book?