Math CS-120: Homework 4

1. Let a function $f : [a, b] \to \mathbb{R}$ be bounded. Prove that f is continuous at a point $x_o \in [a, b]$ if and only if $\omega_f(x_o) = 0$. Recall our definitions:

$$\omega_f(x_o) = \lim_{h \to 0^+} \Omega_f((x_o - h, x_o + h) \cap [a, b])$$

where $\Omega_f(T) = \sup\{f(x) - f(y) | x \in T, y \in T\}$ for any $T \subseteq S$.

2. Recall the function $g : [0, 1] \to \mathbb{R}$ defined as 0 on the irrational numbers in $[0, 1]$ and as $\frac{1}{n}$ on the rational number $\frac{m}{n}$ (where the rational number is written lowest terms, with $n > 0$). Compute $\omega_f(x)$ for each $x \in [0, 1]$. Then, what does Lebesgue's condition tell you about the function g?

3. Chapter 4, exercise 1.7.

4. Chapter 4, exercise 1.8.

5. Show that if A and B are contented sets in $\mathbb{R}^n$, then $A \cup B$ and $A \cap B$ are contented with $v(A \cup B) + v(A \cap B) = v(A) + v(B)$.