

Math 8: Midterm Solution

1)

- a) $A \cup B = \{x : x \in A \vee x \in B\}$
- b) x is rational if $x = \frac{p}{q}$ where $p, q \in \mathbb{Z}$ and $q \neq 0$
- c) An integer p is prime if $p \geq 2$ and for every integer $d \geq 0$, if d divides p , then $d = 1$ or $d = p$

2)

- a) All dogs are blue and some birds can fly
- b) p is prime and p is even and p is not equal to 2, or p is equal to 2, and p is not prime or p is not even
- c) You either have more than one soulmate or you have no soulmate

3)

- a) False. If $a = 1$, $b = 2$, and $c = 4$, then a divides b , b divides c , but b does not divide $a + c$ (2 does not divide 5)
- b) True. $x \in (A - C) \cap (B - C) \Leftrightarrow x \in (A - C) \wedge x \in (B - C) \Leftrightarrow (x \in A \wedge x \notin C) \wedge (x \in B \wedge x \notin C) \Leftrightarrow (x \in A \wedge x \in B) \wedge (x \in C \wedge x \in C) \Leftrightarrow x \in (A \cap B) \wedge x \notin C \Leftrightarrow x \in (A \cap B) - C$
- c) True. If $a - b$ is odd, then $a - b = 2k + 1$ for some integer k . So $a + b = a - b + 2b = 2k + 1 + 2b = 2(k + b) + 1$. Therefore $a + b$ is odd.
- d) False. A is an element of the power set of A , not a subset. Let $A = \{1\}$.

4)

- a) Assume $a + x$ is rational. Then $a + x = \frac{p}{q}$ where $p, q \in \mathbb{Z}$ and $q \neq 0$. Also $a = \frac{r}{s}$ where $r, s \in \mathbb{Z}$ and $s \neq 0$. So $x = \frac{p}{q} - \frac{r}{s} = \frac{ps - qr}{qs}$ and $qs \neq 0$. So x is rational. So we have proved the contrapositive, so by contraposition the claim is true.
- b) For $n = 1$, $5 \geq 5$. Assume that $5^n \geq 1 + 4n$. Then $5^{n+1} = 5 * 5^n \geq 5 * (1 + 4n) = 5 + 20n \geq 1 + 4 + 4n = 1 + 4(n + 1)$

Extra Credit:

Prove by contraposition. If $T \subseteq \mathbb{N}$ and T does not have a smallest element, then $T = \emptyset$. $1 \in T$ because T does not have a smallest element. Assume $1, \dots, n$ are not in T . If $n+1 \in T$ then $n+1$ would be the smallest element of T . But T has no smallest element, so $n+1 \notin T$. Therefore T contains no natural numbers. So $T = \emptyset$.