

Math 8: Homework #3 Solution

1.6 - 1 b,e

- b) Let $m = 1$ and $n = -1$. Then $15(1) + (-1)12 = 3$.
- e) Suppose there are integers m , n , and t such that $15m + 16n = t$. Let $r = 5m$ and $s = 2n$, which are both integers. Then the above equation becomes $3r + 8s = t$.

1.6 - 5 b,e

- b) Take any $x \in \mathbb{R}$. Let $y = -x$. Then $x + y = x + (-x) = x - x = 0$.
- e) Suppose a , b , c , and d are integers, and a divides $b - c$ and a divides $c - d$. Then $b - c = aq$ for some integer q , and $c - d = ar$ for some integer r . So $d = c - ar$. Therefore, $b - d = b - (c - ar) = b - c + ar = aq + ar = a(q + r)$. And $q + r$ is an integer, so a divides $b - d$.

1.6 - 8 c

- c) In this problem, the dude says that a divides b , so $a = bk$ for some integer k . This is wrong. If a divides b , then $ak = b$ for some integer k . So then $(ak)^n = a^n k^n = b^n$ so since k^n is an integer, a^n divides b^n . I would give this proof a C . Small error, proof mostly right.

2.1 - 1 b,e

- b) $\{x \in \mathbb{Z} : x < 17\}$
- e) $\{x \in \mathbb{R} : -5 \leq x \leq -1\}$

2.1 - 6

- a) $\mathcal{P}(X) = \{\emptyset, \{0\}, \{\square\}, \{\triangle\}, \{0, \triangle\}, \{0, \square\}, \{\triangle, \square\}, X\}$
- b) $\mathcal{P}(X) = \{\emptyset, X, \{S\}, \{\{S\}\}\}$
- c) $\mathcal{P}(X) = \{\emptyset, X, \{\emptyset\}, \{\{a\}\}, \{\{b\}\}, \{\{a, b\}\}, \{\emptyset, \{a\}\}, \{\emptyset, \{b\}\}, \{\emptyset, \{a, b\}\}, \{\{a\}, \{b\}\}, \{\{a\}, \{a, b\}\}, \{\{b\}, \{a, b\}\}, X\}$
- d) $\mathcal{P}(X) = \{\emptyset, X, \{1\}, \{\{2, \{3\}\}\}\}$
- e) $\mathcal{P}(X) = \{\emptyset, X, \{1\}, \{2\}, \{3\}, \{4\}, \{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, X\}$

2.1 - 8 b, c

- b) If $A = B$, then $A \subseteq B$, and $\mathcal{P}(B) \subseteq \mathcal{P}(A)$. So for instance, let $A = B = \emptyset$, then $\mathcal{P}(A) = \mathcal{P}(B) = \{\emptyset\}$.
- c) The smallest possible set is the empty set, whose power set is $\{\emptyset\}$ (i.e. a one element set, not the empty set). So this is not possible.

2.2 - 1 b,d,f,h,j

- b) $A \cap B = \emptyset$
- d) $B - C = \{0, 6\}$ so $A - (B - C) = \{1, 3, 5, 7, 9\}$
- f) $C \cap D = \{1, 2, 5, 7, 8\}$ so $A \cap (C \cap D) = \{1, 5, 7\}$
- h) $B \cup C = \{0, 1, 2, 4, 5, 6, 7, 8\}$ so $A \cap (B \cup C) = \{1, 5, 7\}$
- j) $A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $C \cap D = \{1, 2, 5, 7, 8\}$, so $(A \cup B) - (C \cap D) = \{0, 3, 4, 6, 9\}$

2.2 - 13 b

- b) Take any $S \in \mathcal{P}(A) \cup \mathcal{P}(B)$. Then $S \in \mathcal{P}(A)$ or $S \in \mathcal{P}(B)$ (or both). Then $S \subseteq A$ or $S \subseteq B$. So $S \subseteq (A \cup B)$. Therefore $S \in \mathcal{P}(A \cup B)$. Thus, $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.
- c) If A and B are disjoint (i.e. $A \cap B = \emptyset$), then equality doesn't hold. Try $A = 1$ and $B = 2$. If $A \subseteq B$, then the equality will hold since then $\mathcal{P}(A \cup B) = \mathcal{P}(B)$ and $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(B)$.